Solutions

2.1

- **13, 29, 37**
- 1 is not prime; $9 = 3 \times 3$, $21 = 3 \times 7$ and $33 = 3 \times 11$

2.2

- indices first: $3^2 = 9$; then $4 \times 9 = 36$ and $6 \div 2 = 3$
- $5 + 36 - 3 = \mathbf{38}$

2.3

- factor tree: $60 = 2 \times 30 = 2 \times 2 \times 15 = 2 \times 2 \times 3 \times 5$
- $60 = \mathbf{2^2 \times 3 \times 5}$

2.4

- $48 = 2^4 \times 3$ and $60 = 2^2 \times 3 \times 5$
- shared primes, lowest powers: HCF $= 2^2 \times 3 = \mathbf{12}$

2.5

- $18 = 2 \times 3^2$ and $24 = 2^3 \times 3$
- every prime, highest powers: LCM $= 2^3 \times 3^2 = 8 \times 9 = \mathbf{72}$

2.6

- $2 + 6 \times (5 - 3) = 2 + 6 \times 2 = 2 + 12 = 14$

3.1

(a)

- $126 = 2 \times 63 = 2 \times 7 \times 9$
- $126 = \mathbf{2 \times 3^2 \times 7}$

(b)

- shared primes 2 and 3, lowest powers: HCF $= 2 \times 3 = \mathbf{6}$
- every prime, highest powers: LCM $= 2^3 \times 3^2 \times 5 \times 7 = \mathbf{2520}$

3.2

- a counterexample with both values: 4 and 6 have LCM $= 12$, but $4 \times 6 = 24$
- so the LCM is not always the product: 4 and 6 share the factor 2, which the product counts twice

3.3

(a)

- "as many identical bunches as possible, using everything" means the **HCF**
- $84 = 2^2 \times 3 \times 7$ and $126 = 2 \times 3^2 \times 7$
- HCF $= 2 \times 3 \times 7 = \mathbf{42}$ bunches

(b)

- roses: $84 \div 42 = 2$; tulips: $126 \div 42 = 3$, so **2 roses and 3 tulips**

3.4

- "next leave together" means the **LCM** of 15 and 25
- $15 = 3 \times 5$ and $25 = 5^2$, so LCM $= 3 \times 5^2 = 75$ minutes
- 75 minutes after 08:00 is **09:15**

3.5

- use HCF $\times$ LCM $= n \times 36$
- $12 \times 180 = 36n$, so $n = \dfrac{12 \times 180}{36} = \mathbf{60}$
- check: $60 = 2^2 \times 3 \times 5$ and $36 = 2^2 \times 3^2$ give HCF $= 2^2 \times 3 = 12$ and LCM $= 2^2 \times 3^2 \times 5 = 180$