

4.3.5 Spatial Triangles

Name: _____ Class: _____ Date: _____

KEY WORDS projection

<!-- Session 2 native-skill companion -->

Objective

Practise spatial triangles within the labelled native lessons. Public original tasks; locally authored answer guidance.

1 Key knowledge and worked examples

■ Right triangles in three-dimensional shapes · Higher

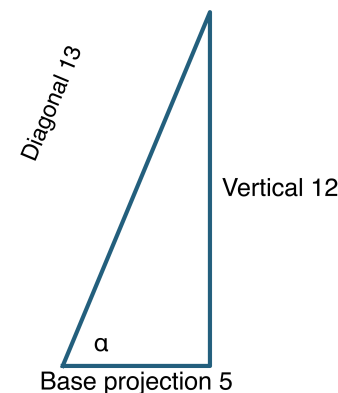
Identify a plane containing the wanted length or angle. Calculate a base diagonal first, then combine it with the perpendicular height. For an angle between a line and a plane, use the angle between the line and its orthogonal projection onto that plane. Mark right angles; general-triangle rules are used only when the selected triangle is not right-angled.

$$d = \sqrt{l^2 + w^2 + h^2}, \quad \tan \alpha = \frac{h}{\sqrt{l^2 + w^2}}$$

A cuboid of width 3, depth 4 and height 12 has base diagonal $\sqrt{9+16}=5$ and space diagonal $\sqrt{25+144}=13$. The angle α of the space diagonal to the horizontal base satisfies $\tan \alpha = 12/5$, giving about 67.4° . Its sine is $12/13$; its cosine is $5/13$. In a square-based pyramid of side 6 and vertical height 4, the base centre-to-side-midpoint distance is 3, giving face slant height 5. The centre-to-corner distance is $3\sqrt{2}$, giving edge length $\sqrt{18+16}=\sqrt{34}$. The face slant and edge lengths differ because their base projections differ.

Do not combine unrelated lengths as if they met at a right angle. The angle to a plane uses the base projection, not an arbitrary base edge. A pyramid's face slant is different from its sloping edge.

AQA G20 Higher extends right-triangle and, where possible, general-triangle reasoning into 3D. Draw the relevant section triangle separately and name which spatial points it represents.

Cuboid section: first $\sqrt{3^2+4^2}=5$, then $\sqrt{5^2+12^2}=13$ 

Original native-lesson illustration; labels belong to its worked example.

2 Practice

2.1 (Higher) Find the base diagonal of a $3 \times 4 \times 12$ cuboid.

2.2 (Higher) Find its space diagonal.

2.3 (Higher) Find tangent of the space-diagonal angle to the base.

3 Exam-style questions

3.1 Decide whether this claim is true. Justify with a concrete calculation or counterexample: A cuboid's space diagonal can always be found from just two of its three edge lengths.

Solutions

2.1 $\sqrt{3^2+4^2}=5$.

2.2 $\sqrt{5^2+12^2}=13$.

2.3 Opposite 12 divided by base projection 5.

3.1 False. Edges 3,4,12 give diagonal $\sqrt{9+16+144}=13$. Using only 3 and 4 gives the face diagonal 5, not the space diagonal.

Local review guidance: show the chosen method; give the checked result; explain the condition or failed shortcut. This is authored classroom guidance.