

4.3.4 General Triangles

Name: _____ Class: _____ Date: _____

KEY WORDS included angle

<!-- Session 2 native-skill companion -->

Objective

Practise general triangles within the labelled native lessons. Public original tasks; locally authored answer guidance.

1 Key knowledge and worked examples

■ Sine rule, cosine rule and triangle area · Higher

Sine rule pairs opposite sides/angles: $a/\sin A = b/\sin B = c/\sin C$. Cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$ uses the angle opposite a . Area is $ab \sin C / 2$ when C lies between a and b . Choose the rule from the known information. An inverse sine may give an acute angle and an obtuse supplement; check the angle sum and supplied sides before accepting either.

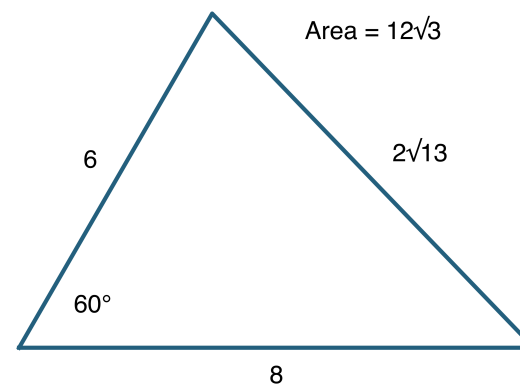
$$a^2 = b^2 + c^2 - 2bc \cos A, \quad A_{\Delta} = \frac{1}{2} ab \sin C$$

With sides 6 and 8 enclosing 60° , $c^2 = 36 + 64 - 96 \times (1/2) = 52$, hence $c = 2\sqrt{13}$. Its area is $(1/2) \times 6 \times 8 \times \sin 60^\circ = 12\sqrt{3}$. For $a=4$ opposite $A=30^\circ$ and $B=45^\circ$, $b = 4 \sin 45^\circ / \sin 30^\circ = 4\sqrt{2}$. If sides $a=7, b=5, c=6$, $\cos A = (25 + 36 - 49) / (2 \times 5 \times 6) = 1/5$, so $A \approx 78.5^\circ$. To find an angle from area 12 with enclosing sides 6 and 8, $\sin C = 24/48 = 1/2$; C could be 30° or 150° until the remaining data selects a shape. Label opposite pairs and check triangle inequalities to reject impossible side combinations.

Do not pair a side with its adjacent angle in the sine rule. The area angle must be included. A calculator's first inverse-sine answer need not be the only possible triangle.

AQA G22/G23 Higher includes unknown sides/angles and areas of general triangles. Write the chosen rule before substitution and state any second possible configuration.

Two sides and their included angle



Original native-lesson illustration; labels belong to its worked example.

2 Practice

2.1 (Higher) For sides 6, 8 and included 60° , find the square of the third side.

2.2 (Higher) For sides 7, 5, 6, find cosine of the angle opposite 7.

2.3 (Higher) For area 12 and enclosing sides 6, 8, find sine of the included angle.

3 Exam-style questions

3.1 Decide whether this claim is true. Justify with a concrete calculation or counterexample: The sine rule pairs each side with any angle in the triangle.

Solutions

2.1 $36+64-96\times\cos60^\circ=52$.

2.2 $(25+36-49)/(2\times5\times6)=12/60$.

2.3 $\sin C=2\times12/(6\times8)=1/2$.

3.1 False. In a 30° – 60° – 90° triangle with opposite sides 1, $\sqrt{3}$,2, the correct ratios are $1/\sin30^\circ=2$ and $2/\sin90^\circ=2$. Pairing side 1 with 60° instead gives $2/\sqrt{3}$, which is different.

Local review guidance: show the chosen method; give the checked result; explain the condition or failed shortcut. This is authored classroom guidance.