

2.5.3 Equation Iteration

Name: _____ Class: _____ Date: _____

KEY WORDS iteration

<!-- Session 2 native-skill companion -->

Objective

Practise equation iteration within the labelled native lessons. Public original tasks; locally authored answer guidance.

1 Key knowledge and worked examples

■ Numerical iteration for equation roots · Higher

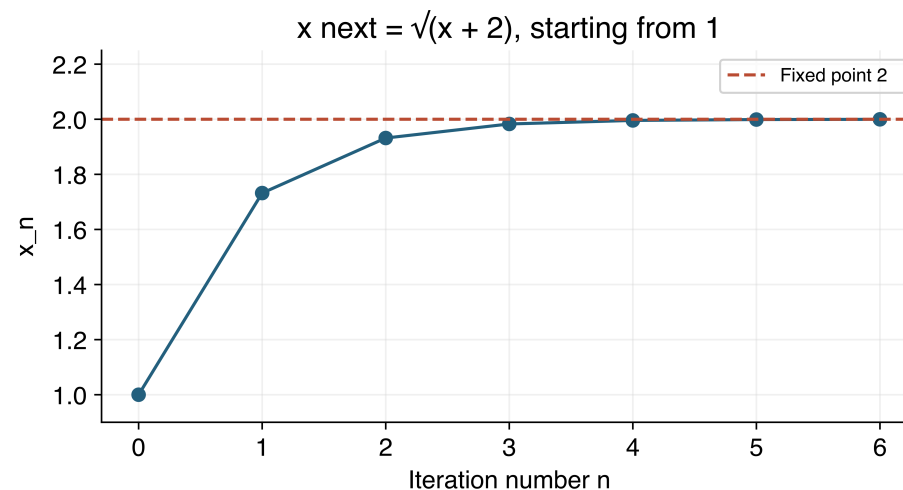
Rearrange an equation as $x=g(x)$, choose x_0 and use $x_{(n+1)}=g(x_n)$. Record sufficient working precision. A stable-looking sequence must still be checked in the original equation; not every rearrangement converges. A sign change across continuous inputs can check a rounded root.

$$x_{n+1} = \sqrt{x_n + 2}, \quad x_0 = 1$$

For $x^2-x-2=0$, use $x_{\text{next}}=\sqrt{x+2}$ with $x_0=1$: $x_1=\sqrt{3}\approx 1.73205$, $x_2\approx 1.93185$, $x_3\approx 1.98289$. The positive fixed point is 2 because $2=\sqrt{4}$ and $2^2-2-2=0$. The rearrangement only seeks a nonnegative root; the original equation also has root -1. Using $x_{\text{next}}=x^2-2$ from 3 instead gives 7 then 47, so a different rearrangement can diverge.

Use the previous iterate, not the starting value every time. Keep more digits than the final answer. A sign change needs continuity; a jump across a vertical asymptote does not guarantee a root.

AQA A20 Higher uses numerical iteration and suffix notation. Use the calculator to follow the stated rule, then report the requested rounding with a check. Newton derivatives are outside this GCSE lesson.



Original native-lesson illustration; labels belong to its worked example.

2 Practice

2.1 (Higher) Find x_1 from $x_{\text{next}}=\sqrt{x+2}$, $x_0=1$.

2.2 (Higher) Find the positive fixed point.

2.3 (Higher) Find the original-equation residual at $x=2$.

3 Exam-style questions

3.1 Decide whether this claim is true. Justify with a concrete calculation or counterexample: Every rearrangement of an equation converges from every starting value.

Solutions

2.1 $\sqrt{1+2}=\sqrt{3}$.

2.2 Solve $L^2-L-2=0$ and retain the nonnegative solution.

2.3 $2^2-2-2=0$.

3.1 False. The equation $x=2$ can be rewritten $x=2x-2$. Iteration $x_{\text{next}}=2x-2$ from $x_0=3$ gives 4,6,10,... and diverges, although 2 is a fixed point.

Local review guidance: show the chosen method; give the checked result; explain the condition or failed shortcut. This is authored classroom guidance.