

2.4.3 Functions

Name: _____ Class: _____ Date: _____

KEY WORDS domain

<!-- Session 2 native-skill companion -->

Objective

Practise functions within the labelled native lessons. Public original tasks; locally authored answer guidance.

1 Key knowledge and worked examples

■ Domains, inverses and composition · Higher

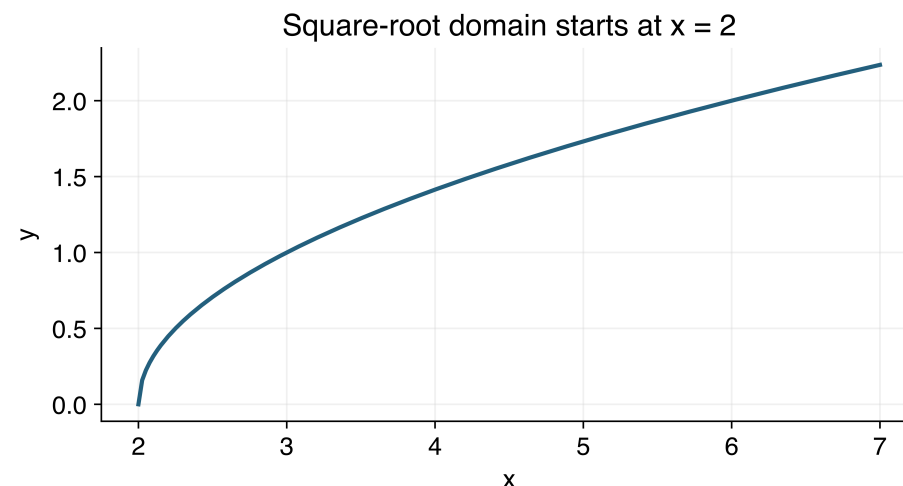
State the domain and range. For an inverse, first ensure the function is one-to-one on its domain. Composition fg means apply g first, then f ; the intermediate output must be an allowed input to f .

$$f(g(x)) = (f \circ g)(x), \quad f^{-1}(f(x)) = x$$

For $f(x)=\sqrt{x-2}$, $x \geq 2$ and the range is $y \geq 0$. From $y=\sqrt{x-2}$, $x=y^2+2$. Thus $f^{-1}(x)=x^2+2$ with $x \geq 0$. For $g(x)=x+3$, $fg(1)=f(4)=\sqrt{2}$.

Squaring can introduce extraneous solutions. Restricting a parabola's domain is essential before claiming an inverse. A horizontal translation inside f has the opposite sign to the graph's movement.

Check $f(f^{-1}(x))=x$ on the inverse domain. Use a sketch to test whether a horizontal line meets the original graph more than once.



Original native-lesson illustration; labels belong to its worked example.

2 Practice

2.1 (Higher) For $f(x)=2x+3$, find $f^{-1}(11)$.

2.2 (Higher) For $f(x)=2x+3$ and $g(x)=x^2$, find $fg(4)$.

2.3 (Higher) For $f(x)=\sqrt{x-2}$, find $f(11)$.

3 Exam-style questions

3.1 Decide whether this claim is true. Justify with a concrete calculation or counterexample: Every quadratic function on all real numbers has an inverse function.

Solutions

2.1 Solve $2x+3=11$, giving $x=4$.

2.2 Apply g first: $g(4)=16$; then $f(16)=2 \times 16+3=35$.

2.3 Substitute into the square root: $\sqrt{(11-2)}=\sqrt{9}=3$.

3.1 False. For $f(x)=x^2$, $f(-1)=f(1)=1$, so the inverse would have two outputs for input 1. Restricting the domain, for example $x \geq 0$, makes an inverse function possible.

Local review guidance: show the chosen method; give the checked result; explain the condition or failed shortcut. This is authored classroom guidance.