

2.3.3 Quadratic Sequences

Name: _____ Class: _____ Date: _____

KEY WORDS second difference

<!-- Session 2 native-skill companion -->

Objective

Practise quadratic sequences within the labelled native lessons. Public original tasks; locally authored answer guidance.

1 Key knowledge and worked examples

■ Quadratic nth terms and surd-ratio progressions · Higher

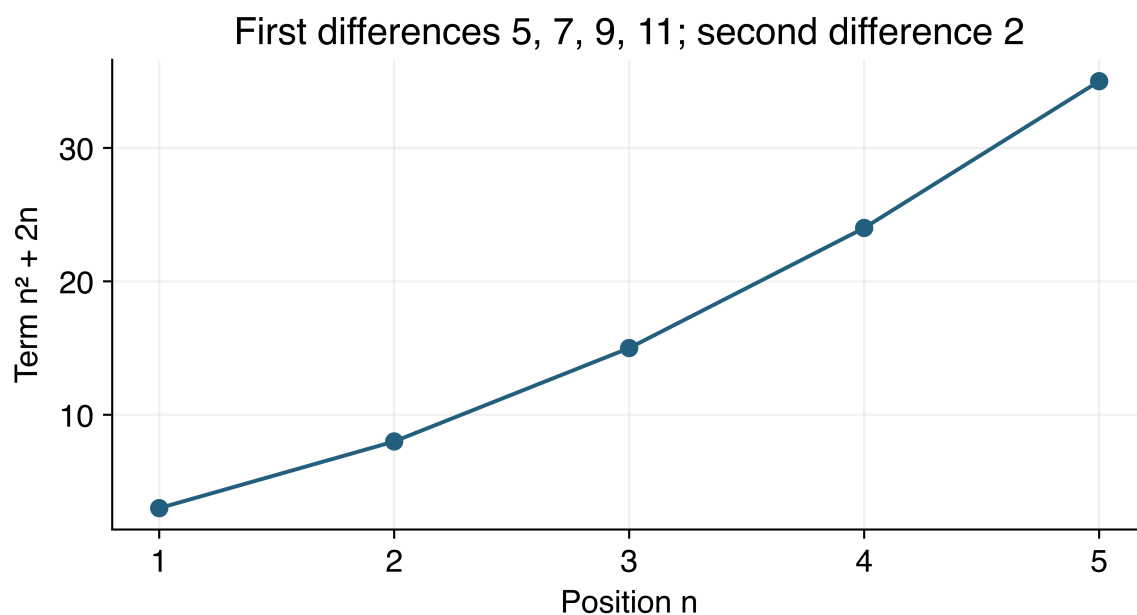
A constant second difference signals a quadratic rule $an^2 + bn + c$. Its value is $2a$. Subtract an^2 from the terms; fit the remaining linear rule and test at several positions. A geometric sequence with a surd ratio still multiplies by the same exact factor; keep root values exact.

$$u_n = n^2 + 2n, \quad \Delta^2 u_n = 2$$

For 3,8,15,24, first differences are 5,7,9 and second difference 2, so $a = 1$. Subtract n^2 at $n=1,2,3,4$ to get 2,4,6,8= $2n$. Thus $u_n = n^2 + 2n$ and $u_5 = 35$. For $1, \sqrt{2}, 2, 2\sqrt{2}, \dots$ the ratio is $\sqrt{2}$ and $u_n = (\sqrt{2})^{(n-1)}$. Check $u_3 = 2$ rather than rounding the root repeatedly.

The second difference equals $2a$, not a . A quadratic rule must be checked against the first terms; there can be a nonzero constant c . A finite pattern alone does not prove a unique rule without the stated sequence family.

AQA A25 Higher derives quadratic nth terms; A24 permits surd-ratio sequences. Foundation recognises and generates quadratic patterns without this general derivation.



Original native-lesson illustration; labels belong to its worked example.

2 Practice

2.1 (Higher) Find a when the second difference is 2.

2.2 (Higher) Find u_5 for $u_n = n^2 + 2n$.

2.3 (Higher) Find the third term of the geometric sequence starting 1 with ratio $\sqrt{2}$.

3 Exam-style questions

3.1 Decide whether this claim is true. Justify with a concrete calculation or counterexample: The coefficient of n^2 always equals the second difference.

Solutions

2.1 $2a=2$, so $a = 1$.

2.2 $25+10=35$.

2.3 $1 \times \sqrt{2} \times \sqrt{2} = 2$.

3.1 False. For n^2 the terms 1,4,9 have first differences 3,5 and second difference 2. The coefficient is 1, half the constant second difference.

Local review guidance: show the chosen method; give the checked result; explain the condition or failed shortcut. This is authored classroom guidance.