

Exercise walk-through

5.2

Tree diagrams and sets - HT (P7–P9)

AQA · GCSE · Mathematics

1. Unit 3.5 at a glance

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Foundation rows P1-P9 in three tiers. Additional foundation: empirical unbiased samples tending toward theory (P5), tree diagrams (P6) and independent plus DEPENDENT combined events with their assumptions (P8). Higher only: conditional probability through expected frequencies with two-way tables, trees and Venn diagrams (P9).

Must-know: probabilities on the 0-to-1 scale; exhaustive mutually exclusive outcomes sum to 1; relative frequency = occurrences over trials; expected count = n times p ; multiply along tree paths, add disjoint paths; $P(A \text{ given } B) = P(A \text{ and } B) \text{ over } P(B)$.

4. Systematic listing and possibility spaces

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Enumerate sets and combinations systematically with tables, grids, Venn diagrams and trees (P6). Possibility spaces for equally likely single and combined experiments (P7): coin and die give 12 ordered outcomes, heads-even is $3/12$; two dice give 36 pairs - total 7 is $6/36$, at least 11 is $3/36$. The 3H Q22 anchor: four different digits from five give $5 \times 4 \times 3 \times 2 = 120$ codes.

Venn regions fill overlap first: 18 cycle, 12 swim, 7 both in a class of 30 gives 11 only-cycling, 5 only-swimming, 23 at least one, 7 neither. The 2H Q9 anchor: both A and B shades the intersection lens only.

5. Combined events and tree diagrams

Combined events and tree diagrams

Multiply along a path, add disjoint paths (P8) - and know when: independence keeps stage denominators fixed. Bag of 3 red 2 blue: with replacement two reds is $\frac{3}{5} \times \frac{3}{5} = \frac{9}{25}$; without, $\frac{3}{5} \times \frac{2}{4} = \frac{3}{10}$, and one of each is $\frac{3}{5} \times \frac{2}{4} + \frac{2}{5} \times \frac{3}{4} = \frac{3}{5}$.

The 3H Q12(b) anchor: failing with probability 0.05 independently each day, at least one failure in two days is $1 - 0.95 \times 0.95 = 0.0975$ - the complement route beats summing three paths. Two disjoint events with positive probability are NOT independent.

Exercises 2.1·2.3

2.1 A bag has 3 red and 2 blue counters. One is drawn, REPLACED, and another drawn. Find $P(\text{two red})$. [3]

2.2 The same bag, now WITHOUT replacement. Find $P(\text{two red})$. [3]

2.3 Without replacement, find $P(\text{one of each colour})$. [3]

Exercises 2.1·2.3

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2.1 $\frac{3}{5} \times \frac{3}{5} = \frac{9}{25}$ – replacement keeps both denominators at 5.

2.2 The same bag, now WITHOUT replacement. Find $P(\text{two red})$. [3]

2.2 $\frac{3}{5} \times \frac{2}{4} = \frac{6}{20} = \frac{3}{10}$.

2.3 Without replacement, find $P(\text{one of each colour})$. [3]

2.3 $\frac{3}{5} \times \frac{2}{4} + \frac{2}{5} \times \frac{3}{4} = \frac{6}{20} + \frac{6}{20} = \frac{12}{20} = \frac{3}{5}$ – the two disjoint paths add.

Exercises 2.4·3.1

2.4 (*June 2024 2H Q4(a), adapted.*) A fair spinner has six equal sections labelled 5, 6, 7 or 8, each appearing at least once, and $P(\text{even}) = P(7)$. Work out $P(5)$. [2]

2.5 In a class of 30, 18 cycle, 12 swim and 7 do both. Find $P(\text{swims} \mid \text{cycles})$ and $P(\text{cycles} \mid \text{swims})$. [3]

3.1 (*June 2024 3H Q12(b), adapted.*) A machine fails on a given day with probability 0.05, independently each day. Find the probability that it fails on at least one of two days. [3]

Exercises 2.4·3.1

2.4 (*June 2024 2H Q4(a), adapted.*) A fair spinner has six equal sections labelled 5, 6, 7 or 8, each appearing at least once, and $P(\text{even}) = P(7)$. Work out $P(5)$. [2]

2.4 Six sections with each number at least once and as many 7s as even numbers (the 6s and 8s): sections 5, 5, 6, 7, 7, 8 work, so $P(5) = \frac{2}{6} = \frac{1}{3}$.

2.5 In a class of 30, 18 cycle, 12 swim and 7 do both. Find $P(\text{swims} \mid \text{cycles})$ and $P(\text{cycles} \mid \text{swims})$. [3]

2.5 $P(\text{swims} \mid \text{cycles}) = \frac{7}{18}$ (denominator 18 cyclists); $P(\text{cycles} \mid \text{swims}) = \frac{7}{12}$ (denominator 12 swimmers).

3.1 (*June 2024 3H Q12(b), adapted.*) A machine fails on a given day with probability 0.05, independently each day. Find the probability that it fails on at least one of two days. [3]

3.1 $P(\text{no failure in two days}) = 0.95 \times 0.95 = 0.9025$, so $P(\text{at least one}) = 1 - 0.9025 = 0.0975$.

Exercises 3.2-3.3

3.2 (*Original, expected frequencies.*) A condition affects 20% of a population of 1000. A test is positive for 90% of those with the condition and for 10% of those without it. Work out the probability that a person chosen at random from those testing positive has the condition. [4]

3.3 (*Original.*) $P(A) = 0.4$, $P(B) = 0.3$ and $P(A \cap B) = 0.12$. Are A and B independent? Show working. [3]

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3.2 Expected counts: with the condition 200, of whom $0.9 \times 200 = 180$ test positive; without it 800, of whom $0.1 \times 800 = 80$ test positive. Positives = 260, so $P(\text{condition} \mid \text{positive}) = \frac{180}{260} = \frac{9}{13} \approx 0.692$.

3.3 (*Original.*) $P(A) = 0.4$, $P(B) = 0.3$ and $P(A \cap B) = 0.12$. Are A and B independent? Show working. [3]

3.3 $P(A) \times P(B) = 0.4 \times 0.3 = 0.12 = P(A \cap B)$, so A and B are independent.

Exercise 3.4

3.4 (*Original.*) Two fair dice are rolled. Find $P(\text{total is at least } 11)$.

[2]

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3.4 Three of the 36 ordered pairs give at least 11 – (5,6), (6,5), (6,6) – so $P = \frac{3}{36} = \frac{1}{12}$.

Local review guidance: tree branches labelled before multiplying, second stage adjusted when there is no replacement; the complement route $1 - 0.95^2$ accepted in 3.1; expected frequencies kept whole until the final ratio – the restricted denominator 260, never 1000. This is authored classroom guidance.