

Exercise walk-through

5.2.2

Conditional Counts

AQA · GCSE · Mathematics

29. Venn diagrams, unions and two-way counts · Foundation

Venn diagrams, unions and two-way counts · Foundation

In a class of 30, 18 cycle, 12 swim and 7 do both. Cycling only is 11 and swimming only 5; at least one is $11+7+5=23$, leaving 7 neither. A random student has probability $7/30$ of both and $23/30$ of at least one. The two-way table has cycle-and-swim 7, cycle-not-swim 11, not-cycle-swim 5 and neither 7. The cycle row totals 18, swim column totals 12 and grand total 30. The outcomes both, cycling only, swimming only and neither are mutually exclusive and exhaustive, so their probabilities sum to 1.

36. Conditional probability with tables and expected frequencies · Higher

Conditional probability with tables and expected frequencies · Higher

Using a class of 30 with 18 cyclists, 12 swimmers and 7 doing both, $P(\text{swim given cycle})=7/18$, while $P(\text{cycle given swim})=7/12$. The unconditional swim probability is $12/30=0.4$. For an expected cohort of 1000, suppose 20% have a condition; a test is positive for 90% with it and 10% without it. Expected positive counts are 180 from 200 with the condition and 80 from 800 without it. Of 260 positive tests, the conditional proportion with the condition is $180/260=9/13\approx 0.6923$, not 90%. These are stated illustrative model rates, not claims about a real diagnostic test.

43. Reasoning and check

Reasoning and check

Both=15, art only=15, music only=10 and neither=10; their sum is 50.

$P(M | A) = 15/30 = 1/2$ while $P(A | M) = 15/25 = 3/5$. The intersection probability is $15/50 = 3/10$, equal to $(30/50)(25/50) = 3/10$, so these events are independent in this finite model. Their unequal conditional probabilities do not contradict independence, since independence compares each condition with the matching unconditional probability: $P(M) = 1/2$ and $P(A) = 3/5$.

Exercises 2.1·2.3

2.1 (Higher) From cyclists 18 and cyclist swimmers 7, find $P(\text{swim given cycle})$.

2.2 (Higher) Find total expected positives in the illustrative 1000-person model.

2.3 (Higher) Find the numerator of $180/260$ in lowest terms.

3 Exam-style questions

Exercises 2.1·2.3

2.1 (Higher) From cyclists 18 and cyclist swimmers 7, find $P(\text{swim given cycle})$.

2.1 Restrict to 18 cyclists; 7 swim.

2.2 (Higher) Find total expected positives in the illustrative 1000-person model.

2.2 180 true positives plus 80 false positives.

2.3 (Higher) Find the numerator of $180/260$ in lowest terms.

3 Exam-style questions

2.3 Divide numerator and denominator by 20 to get $9/13$.

Exercise 3.1

3.1 Decide whether this claim is true. Justify with a concrete calculation or counterexample: $P(A \text{ given } B)$ always equals $P(B \text{ given } A)$.

Exercise 3.1

3.1 Decide whether this claim is true. Justify with a concrete calculation or counterexample: $P(A \text{ given } B)$ always equals $P(B \text{ given } A)$.

3.1 False. In a group of 100, let $|A|=20$, $|B|=50$ and $|A \cap B|=10$. Then $P(A|B)=10/50=1/5$, but $P(B|A)=10/20=1/2$.

Local review guidance: show the chosen method; give the checked result; explain the condition or failed shortcut. This is authored classroom guidance.