

Exercise walk-through

5.1.99

# Independent transfer: Foundation then Higher

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AQA · GCSE · Mathematics

## 16. Experimental probability, fairness and expected outcomes · Foundation

### Experimental probability, fairness and expected outcomes · Foundation

A spinner lands red 18 times in 60 spins, giving estimated  $P(\text{red}) = 18/60 = 0.3 = 30\%$ . Using this estimate predicts  $0.3 \times 200 = 60$  red results in 200 future spins. If red, blue and green are exhaustive with probabilities 0.3, 0.45 and  $p$ , then  $p = 1 - 0.75 = 0.25$ . A fair coin has theoretical  $P(\text{head}) = 0.5$ ; 100 tosses give expected heads 50, but 48 or 54 is possible. An experiment with 6 heads in 10 tosses estimates 0.6; one with 502 heads in 1000 estimates 0.502. These illustrative runs are not proof that error falls at every stage.

## 27. Venn diagrams, unions and two-way counts · Foundation

### Venn diagrams, unions and two-way counts · Foundation

Place the overlap in a Venn diagram first, then fill the only-regions and neither-region. Union means at least one named set; intersection means both; complement means outside a named set in the stated universal group. Two-way tables classify every observation by one category from each of two variables. Check row, column and grand totals.

## 29. Venn diagrams, unions and two-way counts · Foundation

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In a class of 30, 18 cycle, 12 swim and 7 do both. Cycling only is 11 and swimming only 5; at least one is  $11+7+5=23$ , leaving 7 neither. A random student has probability  $7/30$  of both and  $23/30$  of at least one. The two-way table has cycle-and-swim 7, cycle-not-swim 11, not-cycle-swim 5 and neither 7. The cycle row totals 18, swim column totals 12 and grand total 30. The outcomes both, cycling only, swimming only and neither are mutually exclusive and exhaustive, so their probabilities sum to 1.

## Exercise 2.1

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**2.1** Foundation: A bag has three red and two blue counters. Two are drawn without replacement. Make a complete possibility tree and find the probability of different colours. Explain how the second-stage denominators change.

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**2.1** First probabilities are R: $\frac{3}{5}$ , B: $\frac{2}{5}$ . After R, the second probabilities are R: $\frac{2}{4}$ , B: $\frac{2}{4}$ . After B they are R: $\frac{3}{4}$ , B: $\frac{1}{4}$ . Terminal probabilities RR, RB, BR, BB are  $\frac{6}{20}, \frac{6}{20}$  and sum to one. Different colours are the disjoint RB and BR paths, so probability is  $\frac{6}{20} + \frac{6}{20} = \frac{3}{5}$ . Multiplying stages follows a path; adding combines mutually exclusive paths. Replacement would change the model.

## Exercise 3.1

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**3.1** Higher: In 50 students, 30 study art, 25 study music and 15 study both. Find music given art and art given music. Are the events independent? Reconcile the counts using a two-way table.

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**3.1** Both=15, art only=15, music only=10 and neither=10; their sum is 50.  $P(M | A) = 15/30 = 1/2$  while  $P(A | M) = 15/25 = 3/5$ . The intersection probability is  $15/50 = 3/10$ , equal to  $(30/50)(25/50) = 3/10$ , so these events are independent in this finite model. Their unequal conditional probabilities do not contradict independence, since independence compares each condition with the matching unconditional probability:  $P(M) = 1/2$  and  $P(A) = 3/5$ .

Local guidance: select and explain the relationship; calculate every requested result; check units, scope, conditions and completeness. These are authored classroom criteria.