

Exercise walk-through

4.5

Properties and congruence and similarity (G4–G5, G18)

AQA · GCSE · Mathematics

1. Unit 3.4 at a glance

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Foundation rows G1-G25 in three tiers. Higher-only additions: negative enlargement scale factors (G7), change and invariance under combinations of transformations (G8), PROVING the standard circle theorems (G10), constructing plans and elevations (G13), spheres/pyramids/cones/frustums with composite solids (G17), the k - k squared- k cubed similarity ladder (G19), right and general triangles in 3D (G20), the sine and cosine rules (G22), half $ab \sin C$ (G23) and geometric vector proofs (G25).

Must-know: polygon interior sum $(n-2) \times 180$ with exterior turn $360/n$; $C = 2 \pi r = \pi d$ and $A = \pi r^2$; arc and sector scale by $\theta/360$; prism = cross-section $\times$ length; lengths k : areas k^2 : volumes k^3 .

4. Congruence, similarity and circle theorems

Congruence, similarity and circle theorems

Congruence by SSS, SAS, ASA, RHS (G5) - the June 2024 2H Q12 one-word reason is SSS; AAA gives similarity only. Proof chains (G6): isosceles ABC with midpoint D of BC gives SSS on ABD and ACD, so base angles ABC and BCA match; Pythagoras follows from four congruent triangles in an $(a+b)$ square: $(a+b)^2 = 4(ab/2) + c^2$.

Circle parts (G9): 2H Q1 names the segment between chord and arc and the sector between two radii. Theorems (G10): centre angle twice the circumference angle (proof via two isosceles triangles on equal radii); cyclic opposites 180; tangent perpendicular to radius; equal tangents from a point; centre-perpendicular bisects a chord; alternate segment. The 1H Q22 anchor: two tangents at 24 degrees give quadrilateral $360 - 90 - 90 - 24 = 156$ at the centre, so the circumference angle is 78.

Check yourself

Check yourself

Notation and angle facts with named reasons; polygon sums derived; quadrilateral properties; constructions and loci with arcs kept; transformations incl. negative factors and invariance under combinations.

Congruence criteria and proof chains; circle parts and the eight theorems applied and proved; perimeters, areas, volumes incl. sectors, frustums and exact π ; plans, elevations and bearings; the $k : k^2 : k^3$ ladder.

Pythagoras and trig in 2D and 3D; exact trig values; sine and cosine rules; half $ab \sin C$; vectors in column form with collinearity proofs.

Exercises 2.1·2.3

2.1 A quadrilateral has four equal sides and its diagonals bisect each other at right angles, but the diagonals are not equal in length. Name the quadrilateral. [2]

2.2 Write down the defining side property of a kite. [2]

2.3 Two triangles have three pairs of equal corresponding sides. Which congruence criterion applies? [2]

Exercises 2.1·2.3

2.1 A quadrilateral has four equal sides and its diagonals bisect each other at right angles, but the diagonals are not equal in length. Name the quadrilateral. [2]

2.1 A rhombus (a square would additionally need equal diagonals and right angles).

2.2 Write down the defining side property of a kite. [2]

2.2 Two pairs of adjacent sides equal (adjacent, not opposite).

2.3 Two triangles have three pairs of equal corresponding sides. Which congruence criterion applies? [2]

2.3 SSS.

Exercises 2.4·2.6

2.4 Explain why SSA is not a congruence criterion. [2]

2.5 A sector of radius 6 cm has angle 120° . Find its area in terms of π . [2]

2.6 A sector of radius 6 cm has area $9\pi \text{ cm}^2$. Find its angle. [3]

Exercises 2.4·2.6

2.4 Explain why SSA is not a congruence criterion. [2]

2.4 Two different triangles can share two sides and a non-included angle – the given data does not fix the triangle, so congruence can fail.

2.5 A sector of radius 6 cm has angle 120° . Find its area in terms of π . [2]

$$\mathbf{2.5} \quad \frac{120}{360} \times \pi \times 6^2 = \frac{1}{3} \times 36\pi = 12\pi \text{ cm}^2.$$

2.6 A sector of radius 6 cm has area $9\pi \text{ cm}^2$. Find its angle. [3]

$$\mathbf{2.6} \quad \text{Fraction} = \frac{9\pi}{36\pi} = \frac{1}{4}, \text{ so } \theta = \frac{1}{4} \times 360 = 90^\circ.$$

Exercises 3.1·3.3

3.1 (*June 2024 2H Q12, adapted.*) Two triangles have three pairs of equal corresponding sides. Write down the one-word congruence reason. [1]

3.2 (*Original, proof.*) Triangle ABC is isosceles with $AB = AC$, and D is the midpoint of BC . Prove that the base angles ABC and BCA are equal. [4]

3.3 (*Original.*) A sector of radius 6 cm has angle 90° . Find its arc length and its perimeter, in terms of π . [3]

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3.1 SSS.

3.2 (*Original, proof.*) Triangle ABC is isosceles with $AB = AC$, and D is the midpoint of BC . Prove that the base angles ABC and BCA are equal. [4]

3.2 Triangles ABD and ACD have $AB = AC$ (given), $BD = DC$ (D is the midpoint of BC) and AD shared, so they are congruent (SSS). In congruent triangles, matching angles are equal: $\angle ABD$ matches $\angle ACD$. Since D lies on BC , $\angle ABD = \angle ABC$ and $\angle ACD = \angle BCA$; hence the base angles ABC and BCA are equal.

3.3 (*Original.*) A sector of radius 6 cm has angle 90° . Find its arc length and its perimeter, in terms of π . [3]

3.3 Arc = $\frac{90}{360} \times 2\pi \times 6 = 3\pi$ cm; perimeter = $3\pi + 6 + 6 = 3\pi + 12$ cm.

Exercise 3.4

3.4 (*Original.*) An annulus has outer radius 5 cm and inner radius 3 cm. Find its area in terms of π . [2]

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$$\mathbf{3.4} \quad \pi(5^2 - 3^2) = \pi(25 - 9) = 16\pi \text{ cm}^2.$$

Local review guidance: names with the defining property, not appearance; congruence letters ordered so matching parts correspond; each of the three pairs stated with its reason (given, midpoint, common side) before the criterion; the $\frac{\theta}{360}$ fraction shown for every sector calculation. This is authored classroom guidance.