

Exercise walk-through

4.4

Transformations, constructions, loci and vectors (G7–G10, G25)

AQA · GCSE · Mathematics

1. Unit 3.4 at a glance

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Foundation rows G1-G25 in three tiers. Higher-only additions: negative enlargement scale factors (G7), change and invariance under combinations of transformations (G8), PROVING the standard circle theorems (G10), constructing plans and elevations (G13), spheres/pyramids/cones/frustums with composite solids (G17), the k - k squared- k cubed similarity ladder (G19), right and general triangles in 3D (G20), the sine and cosine rules (G22), half $ab \sin C$ (G23) and geometric vector proofs (G25).

Must-know: polygon interior sum $(n-2) \times 180$ with exterior turn $360/n$; $C = 2 \pi r = \pi d$ and $A = \pi r^2$; arc and sector scale by $\theta/360$; prism = cross-section $\times$ length; lengths k : areas k^2 : volumes k^3 .

3. Constructions, loci and transformations

Constructions, loci and transformations

Ruler and compass (G2): perpendicular bisector from equal endpoint arcs, perpendicular from or at a point, angle bisector, a 60-degree angle from two equal circles. Loci: equidistant from two points is the perpendicular bisector; fixed distance r is a circle; equidistant from two lines is an angle bisector. Keep the arcs as evidence.

Enlargement image $C + k(P - C)$ (G7): centre $(1,1)$, point $(3,2)$ - $k = 2$ gives $(5,3)$; the Higher $k = -2$ gives $(-3,-1)$, opposite side. Invariance under combinations (G8): isometries preserve lengths and angles; reflections in $x = 0$ then $x = 2$ send $(3,2)$ to $(-3,2)$ then $(7,2)$ - a translation by $(4,0)$.

Check yourself

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Notation and angle facts with named reasons; polygon sums derived; quadrilateral properties; constructions and loci with arcs kept; transformations incl. negative factors and invariance under combinations.

Congruence criteria and proof chains; circle parts and the eight theorems applied and proved; perimeters, areas, volumes incl. sectors, frustums and exact π ; plans, elevations and bearings; the $k : k^2 : k^3$ ladder.

Pythagoras and trig in 2D and 3D; exact trig values; sine and cosine rules; half $ab \sin C$; vectors in column form with collinearity proofs.

Exercises 2.1·2.3

2.1 Reflect the point $(3, 2)$ in the y -axis. [2]

2.2 Rotate the point $(3, 2)$ by 90° anticlockwise about the origin. [2]

2.3 (*Higher.*) Enlarge $P = (3, 2)$ about $C = (1, 1)$ with scale factor 2, and again with scale factor -2 . [3]

Exercises 2.1·2.3

2.1 Reflect the point $(3, 2)$ in the y -axis. [2]

2.1 $(-3, 2)$.

2.2 Rotate the point $(3, 2)$ by 90° anticlockwise about the origin. [2]

2.2 $(-2, 3)$.

2.3 (*Higher.*) Enlarge $P = (3, 2)$ about $C = (1, 1)$ with scale factor 2, and again with scale factor -2 . [3]

2.3 $k = 2$: $C + 2(P - C) = (1, 1) + 2(2, 1) = (5, 3)$. $k = -2$: $(1, 1) - 2(2, 1) = (-3, -1)$, opposite side of the centre.

Exercises 2.4·2.6

2.4 An arc subtends 100° at the centre. Find its angle at the circumference on the same arc. [2]

2.5 One angle of a cyclic quadrilateral is 112° . Find its opposite angle. [2]

2.6 With $\mathbf{a} = (4, 1)$ and $\mathbf{b} = (1, 3)$, find $\mathbf{a} + \mathbf{b}$ and $2\mathbf{a} - \mathbf{b}$. [3]

Exercises 2.4·2.6

2.4 An arc subtends 100° at the centre. Find its angle at the circumference on the same arc. [2]

2.4 $100 \div 2 = 50^\circ$ – angle at the centre is twice the angle at the circumference.

2.5 One angle of a cyclic quadrilateral is 112° . Find its opposite angle. [2]

2.5 $180 - 112 = 68^\circ$ – opposite angles of a cyclic quadrilateral sum to 180° .

2.6 With $\mathbf{a} = (4, 1)$ and $\mathbf{b} = (1, 3)$, find $\mathbf{a} + \mathbf{b}$ and $2\mathbf{a} - \mathbf{b}$. [3]

2.6 $\mathbf{a} + \mathbf{b} = (5, 4)$; $2\mathbf{a} - \mathbf{b} = (8, 2) - (1, 3) = (7, -1)$.

Exercises 3.1·3.3

3.1 (*June 2024 2H Q1, adapted.*) A circle has centre O . Name in one word each: the region between a chord and its arc; the region between two radii and their arc. [2]

3.2 (*June 2024 1H Q22, adapted.*) Two tangents from an external point P touch a circle, centre O , at A and B . Angle $APB = 24^\circ$. The point C lies on the major arc AB . Work out angle ACB . [4]

3.3 (*June 2024 2H Q23, adapted.*) D , E and F are points with $\overrightarrow{CD} = 6\mathbf{a} + \mathbf{b}$, $\overrightarrow{CE} = 14\mathbf{a} - 3\mathbf{b}$ and $\overrightarrow{CF} = 16\mathbf{a} - 4\mathbf{b}$. (a) Find $\overrightarrow{DE}$ and $\overrightarrow{EF}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. (b) Prove that DEF is a straight line. [4]

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3.1 The chord-and-arc region is the **segment**; the radii-and-arc region is the **sector**.

3.2 (*June 2024 1H Q22, adapted.*) Two tangents from an external point P touch a circle, centre O , at A and B . Angle $APB = 24^\circ$. The point C lies on the major arc AB . Work out angle ACB . [4]

3.2 Each tangent is perpendicular to its radius at the contact point, so quadrilateral $OAPB$ has two 90° angles: $\angle AOB = 360 - 90 - 90 - 24 = 156^\circ$. The angle at the centre is twice the angle at the circumference on the same arc: $\angle ACB = 156 \div 2 = 78^\circ$.

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3.3 (a) $\overrightarrow{DE} = (14\mathbf{a} - 3\mathbf{b}) - (6\mathbf{a} + \mathbf{b}) = 8\mathbf{a} - 4\mathbf{b}$; $\overrightarrow{EF} = (16\mathbf{a} - 4\mathbf{b}) - (14\mathbf{a} - 3\mathbf{b}) = 2\mathbf{a} - \mathbf{b}$.
 (b) $\overrightarrow{DE} = 4(2\mathbf{a} - \mathbf{b}) = 4\overrightarrow{EF}$, so DE is parallel to EF ; they share the point E , so DEF is a straight line.

Exercise 3.4

3.4 (*Original, loci.*) A and B are 6 cm apart. Describe the locus of points equidistant from A and B ; a point on it is 5 cm from A – find its perpendicular distance from AB .^[3]

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3.4 The perpendicular bisector of AB ; half of AB is 3 cm, so the distance is $\sqrt{5^2 - 3^2} = 4$ cm (a 3-4-5 triangle).

Local review guidance: full transformation descriptions – centre, factor, line, angle and direction; the quadrilateral angle sum $360 - 90 - 90 - 24$ then halving for the circumference angle in 3.2; $\overrightarrow{DE} = 4\overrightarrow{EF}$ stated WITH the shared point for the collinearity claim; locus named as a construction, not “the middle”. This is authored classroom guidance.