

Exercise walk-through

4.4.6

Vector Geometry Proofs

AQA · GCSE · Mathematics

28. Vectors and transformation geometry · Foundation

Vectors and transformation geometry · Foundation

With $a=(4,1)$ and $b=(1,3)$, $a+b=(5,4)$ and $2a-b=(7,-1)$. From $A=(1,2)$ to $B=(5,5)$, $AB=(4,3)$ and its length is 5. Write each 2D vector as a column with the horizontal component above the vertical component: the translation AB has top entry 4 and bottom entry 3. A negative horizontal entry moves left; a negative vertical entry moves down. Drawing b from the head of a gives the head-to-tail diagram for $a+b$. Subtracting b adds its opposite $-b$.

174. Vector geometry and midpoint arguments · Higher

Vector geometry and midpoint arguments · Higher

Add/subtract components and multiply a vector by a scalar. Position vectors locate points from one origin; displacement $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$ joins two points. A scalar multiple gives parallel directions; to establish collinearity, also connect the displacements to a common point. Midpoints average position vectors. Give a chain of vector equalities with clear start and end points.

176. Vector geometry and midpoint arguments · Higher

Vector geometry and midpoint arguments · Higher

Let $OA=a$ and $OB=b$, with M midpoint of OA and N midpoint of OB . Then $OM=a/2$ and $ON=b/2$, so $MN=b/2-a/2=(b-a)/2=AB/2$. Thus MN is parallel to AB and half as long. For $a=(4,2), b=(2,6)$, $M=(2,1), N=(1,3)$, $AB=(-2,4)$ and $MN=(-1,2)$, verifying the general result. In parallelogram $OACB$ with $OC=a+b$, the midpoint of OC is $(a+b)/2$; the midpoint of AB is the same, so the diagonals bisect each other. If $AP=3AB/2$, P lies on line AB beyond B ; if $AP=AB/2$, P is its midpoint. A parallel vector at another location alone does not prove three specified points collinear.

Exercises 2.1·2.3

2.1 (Higher) For $a=(4,2)$, $b=(2,6)$, find the x-component of AB .

2.2 (Higher) Find the y-component of midpoint M of OA .

2.3 (Higher) Find the y-component of MN for the two midpoint vectors.

3 Exam-style questions

Exercises 2.1·2.3

2.1 (Higher) For $a=(4,2)$, $b=(2,6)$, find the x-component of AB.

2.1 $b-a$ gives $2-4=-2$.

2.2 (Higher) Find the y-component of midpoint M of OA.

2.2 Half of $a_y=2$ is 1.

2.3 (Higher) Find the y-component of MN for the two midpoint vectors.

3 Exam-style questions

2.3 Half of $b_y - a_y = (6-2)/2 = 2$.

Exercise 3.1

3.1 Decide whether this claim is true. Justify with a concrete calculation or counterexample: If two vectors are parallel, any three points used in their diagrams must be collinear.

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3.1 False. AB from $(0,0)$ to $(1,0)$ and CD from $(0,1)$ to $(2,1)$ are parallel, but A,B,C are not collinear. Collinearity requires a shared line, not merely parallel directions.

Local review guidance: show the chosen method; give the checked result; explain the condition or failed shortcut. This is authored classroom guidance.