

Exercise walk-through

4.3

Pythagoras, trigonometry and their graphs (G6, G20–G24)

AQA · GCSE · Mathematics

4. Congruence, similarity and circle theorems

Congruence, similarity and circle theorems

Congruence by SSS, SAS, ASA, RHS (G5) - the June 2024 2H Q12 one-word reason is SSS; AAA gives similarity only. Proof chains (G6): isosceles ABC with midpoint D of BC gives SSS on ABD and ACD, so base angles ABC and BCA match; Pythagoras follows from four congruent triangles in an $(a+b)$ square: $(a+b)^2 = 4(ab/2) + c^2$.

Circle parts (G9): 2H Q1 names the segment between chord and arc and the sector between two radii. Theorems (G10): centre angle twice the circumference angle (proof via two isosceles triangles on equal radii); cyclic opposites 180; tangent perpendicular to radius; equal tangents from a point; centre-perpendicular bisects a chord; alternate segment. The 1H Q22 anchor: two tangents at 24 degrees give quadrilateral $360 - 90 - 90 - 24 = 156$ at the centre, so the circumference angle is 78.

6. Pythagoras and trigonometry

Pythagoras and trigonometry

Right triangles (G20): $a^2 + b^2 = c^2$; sin, cos, tan as opposite, adjacent over hypotenuse. Exact values (G21) from the 1-root-3-2 and 1-1-root-2 triangles: the 1H Q25 anchor $6 \sin 30 + 2 \cos 30 \times 4 \tan 30 = 3 + 4 = 7$ - surds cancel. The 2H Q18 anchor: with hypotenuse 23.7 and x adjacent to 52 degrees, $\cos 52 = x/23.7$ gives $x = 23.7 \cos 52 = 14.6$.

General triangles, Higher (G22-G23): cosine rule with 6 and 8 enclosing 60 gives $c^2 = 52$, $c = 2\sqrt{13}$; area half $ab \sin C$ gives $12\sqrt{3}$; sine rule with $a = 4$, $A = 30$, $B = 45$ gives $b = 4\sqrt{2}$. In 3D isolate the right triangle: a $3 \times 4 \times 12$ cuboid has base diagonal 5, space diagonal 13, angle $\tan = 12/5$, 67.4 degrees.

Check yourself

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Notation and angle facts with named reasons; polygon sums derived; quadrilateral properties; constructions and loci with arcs kept; transformations incl. negative factors and invariance under combinations.

Congruence criteria and proof chains; circle parts and the eight theorems applied and proved; perimeters, areas, volumes incl. sectors, frustums and exact π ; plans, elevations and bearings; the $k : k^2 : k^3$ ladder.

Pythagoras and trig in 2D and 3D; exact trig values; sine and cosine rules; half $ab \sin C$; vectors in column form with collinearity proofs.

Exercises 2.1·2.3

2.1 A ladder footed 3 m from a wall reaches 4 m up it. Find the ladder's length. [2]

2.2 In a right triangle the opposite and adjacent sides of angle θ are 4 and 3. Find θ to 1 decimal place. [3]

2.3 Write down the exact value of $\sin 30^\circ + \cos 60^\circ$. [2]

Exercises 2.1·2.3

2.1 A ladder footed 3 m from a wall reaches 4 m up it. Find the ladder's length. [2]

2.1 $c = \sqrt{3^2 + 4^2} = 5 \text{ m.}$

2.2 In a right triangle the opposite and adjacent sides of angle θ are 4 and 3. Find θ to 1 decimal place. [3]

2.2 $\tan \theta = \frac{4}{3}$, so $\theta = 53.1^\circ$ (1 dp).

2.3 Write down the exact value of $\sin 30^\circ + \cos 60^\circ$. [2]

2.3 $\frac{1}{2} + \frac{1}{2} = 1.$

Exercises 2.4·2.6

2.4 A right triangle has hypotenuse 10 and an angle of 30° . Write down the exact lengths of the two other sides. [3]

2.5 (*Higher.*) Two sides of a triangle, 6 and 8, enclose 60° . Find the third side in surd form. [3]

2.6 (*Higher.*) In a triangle, $a = 4$ is opposite $A = 30^\circ$ and $B = 45^\circ$. Find b in surd form. [3]

Exercises 2.4·2.6

2.4 A right triangle has hypotenuse 10 and an angle of 30° . Write down the exact lengths of the two other sides. [3]

2.4 Opposite $= 10 \sin 30^\circ = 5$; adjacent $= 10 \cos 30^\circ = 5\sqrt{3}$.

2.5 (*Higher.*) Two sides of a triangle, 6 and 8, enclose 60° . Find the third side in surd form. [3]

2.5 $c^2 = 36 + 64 - 2(6)(8) \cos 60^\circ = 100 - 48 = 52$, so $c = 2\sqrt{13}$.

2.6 (*Higher.*) In a triangle, $a = 4$ is opposite $A = 30^\circ$ and $B = 45^\circ$. Find b in surd form. [3]

2.6 $b = \frac{4 \sin 45^\circ}{\sin 30^\circ} = \frac{4 \times \frac{\sqrt{2}}{2}}{\frac{1}{2}} = 4\sqrt{2}$.

Exercises 3.1·3.3

3.1 (*June 2024 2H Q18, adapted.*) In a right triangle the hypotenuse is 23.7 and x is the side adjacent to the marked angle 52° . Work out x to 1 decimal place. [3]

3.2 (*June 2024 1H Q25, adapted.*) Show that the value of $6 \sin 30^\circ + 2 \cos 30^\circ \times 4 \tan 30^\circ$ is an integer. [3]

3.3 (*Original, Higher 3D.*) A cuboid has width 3 cm, depth 4 cm and height 12 cm. Work out the length of its space diagonal. [4]

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3.1 (*June 2024 2H Q18, adapted.*) In a right triangle the hypotenuse is 23.7 and x is the side adjacent to the marked angle 52° . Work out x to 1 decimal place. [3]

3.1 $\cos 52^\circ = \frac{x}{23.7}$, so $x = 23.7 \cos 52^\circ = 14.6$ (1 dp).

3.2 (*June 2024 1H Q25, adapted.*) Show that the value of $6 \sin 30^\circ + 2 \cos 30^\circ \times 4 \tan 30^\circ$ is an integer. [3]

3.2 $6 \times \frac{1}{2} + \sqrt{3} \times \frac{4}{\sqrt{3}} = 3 + 4 = 7$ – the surds cancel, so the value is the integer 7.

3.3 (*Original, Higher 3D.*) A cuboid has width 3 cm, depth 4 cm and height 12 cm. Work out the length of its space diagonal. [4]

3.3 Base diagonal $= \sqrt{3^2 + 4^2} = 5$; space diagonal $= \sqrt{5^2 + 12^2} = 13$ cm (equivalently $\sqrt{9 + 16 + 144} = 13$).

Exercise 3.4

3.4 (*Original.*) A translation maps $A(1, 2)$ onto $B(5, 5)$. Write the translation as a column vector and find its length. [2]

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3.4 $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$; length $= \sqrt{4^2 + 3^2} = 5$.

Local review guidance: the correct ratio or rule chosen and named before substitution; surd or exact- π -style forms kept exact where asked; the two-stage diagonal, not $\sqrt{3 + 4 + 12}$; column-vector order x above y . This is authored classroom guidance.