

Exercise walk-through

4.3.5

Spatial Triangles

AQA · GCSE · Mathematics

74. Triangle congruence and geometric proof · Foundation

Triangle congruence and geometric proof · Foundation

For an isosceles triangle ABC with $AB=AC$, let D be the midpoint of BC . Triangles ABD and ACD have $AB=AC$, $BD=DC$ and shared AD , so SSS gives congruence. Matching base angles ABC and BCA are therefore equal; the two angles at D are equal and form 180° , so each is 90° . To derive Pythagoras, arrange four congruent right triangles with legs a, b around a tilted square of side c inside a square of side $a+b$. Area gives $(a+b)^2 = 4(ab/2) + c^2$, hence $a^2 + b^2 = c^2$. For $a=3, b=4$, $c^2 = 9 + 16 = 25$, so $c=5$. These arguments establish results independently of a scale drawing.

158. Exact trigonometric values from special triangles · Foundation

Exact trigonometric values from special triangles · Foundation

For angles $0^\circ, 30^\circ, 45^\circ, 60^\circ, 90^\circ$, sine values are $0, 1/2, \sqrt{2}/2, \sqrt{3}/2, 1$; cosine values are $1, \sqrt{3}/2, \sqrt{2}/2, 1/2, 0$. For $0^\circ, 30^\circ, 45^\circ, 60^\circ$, tangent values are $0, 1/\sqrt{3}, 1, \sqrt{3}$. The side 1 opposite 30° in the bisected equilateral triangle gives $\sin 30^\circ = 1/2$; the adjacent $\sqrt{3}$ gives $\cos 30^\circ = \sqrt{3}/2$ and $\tan 30^\circ = 1/\sqrt{3}$. In the isosceles right triangle, $\sin 45^\circ = \cos 45^\circ = 1/\sqrt{2} = \sqrt{2}/2$. A right triangle with hypotenuse 10 and angle 30° has opposite side 5 and adjacent side $5\sqrt{3}$. Since $\sin 90^\circ = 1$ and $\cos 90^\circ = 0$, tangent at 90° would divide by zero.

170. Right triangles in three-dimensional shapes · Higher

Right triangles in three-dimensional shapes · Higher

A cuboid of width 3, depth 4 and height 12 has base diagonal $\sqrt{9+16}=5$ and space diagonal $\sqrt{25+144}=13$. The angle α of the space diagonal to the horizontal base satisfies $\tan\alpha=12/5$, giving about 67.4° . Its sine is $12/13$; its cosine is $5/13$. In a square-based pyramid of side 6 and vertical height 4, the base centre-to-side-midpoint distance is 3, giving face slant height 5. The centre-to-corner distance is $3\sqrt{2}$, giving edge length $\sqrt{18+16}=\sqrt{34}$. The face slant and edge lengths differ because their base projections differ.

Exercises 2.1·2.3

2.1 (Higher) Find the base diagonal of a $3 \times 4 \times 12$ cuboid.

2.2 (Higher) Find its space diagonal.

2.3 (Higher) Find tangent of the space-diagonal angle to the base.

3 Exam-style questions

Exercises 2.1·2.3

2.1 (Higher) Find the base diagonal of a $3 \times 4 \times 12$ cuboid.

2.1 $\sqrt{3^2+4^2}=5$.

2.2 (Higher) Find its space diagonal.

2.2 $\sqrt{5^2+12^2}=13$.

2.3 (Higher) Find tangent of the space-diagonal angle to the base.

3 Exam-style questions

2.3 Opposite 12 divided by base projection 5.

Exercise 3.1

3.1 Decide whether this claim is true. Justify with a concrete calculation or counterexample: A cuboid's space diagonal can always be found from just two of its three edge lengths.

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3.1 False. Edges 3,4,12 give diagonal $\sqrt{9+16+144}=13$. Using only 3 and 4 gives the face diagonal 5, not the space diagonal.

Local review guidance: show the chosen method; give the checked result; explain the condition or failed shortcut. This is authored classroom guidance.