

Exercise walk-through

4.2

Perimeter, area and volume, including circles (G14–G17, G19)

AQA · GCSE · Mathematics

1. Unit 3.4 at a glance

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Foundation rows G1-G25 in three tiers. Higher-only additions: negative enlargement scale factors (G7), change and invariance under combinations of transformations (G8), PROVING the standard circle theorems (G10), constructing plans and elevations (G13), spheres/pyramids/cones/frustums with composite solids (G17), the k - k squared- k cubed similarity ladder (G19), right and general triangles in 3D (G20), the sine and cosine rules (G22), half $ab \sin C$ (G23) and geometric vector proofs (G25).

Must-know: polygon interior sum $(n-2) \times 180$ with exterior turn $360/n$; $C = 2 \pi r = \pi d$ and $A = \pi r^2$; arc and sector scale by $\theta/360$; prism = cross-section $\times$ length; lengths k : areas k^2 : volumes k^3 .

5. Mensuration: perimeter, area and volume

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Plane and solid (G14-G17): triangle half bh , trapezium half $(a+b)h$, prism cross-section $\times$ length, cylinder $\pi r^2 h$, cone one-third $\pi r^2 h$ with slant from Pythagoras, sphere four-thirds πr^3 . Exact π answers are asked for: radius 12 at 60 degrees gives arc $60/360 \times 24\pi = 4\pi$ (the 1H Q12 anchor); a cone $r = 3$, $h = 4$ has slant 5 and volume 12π ; its frustum against a similar 6-8 cone keeps $96\pi - 12\pi = 84\pi$.

Solids and views (G12-G13): a $4 \times 3 \times 2$ cuboid has plan 4×3 , front 4×2 , side 3×2 ; Higher constructs these too. Bearings (G15): three figures clockwise from north; 070 reverses to 250. Similarity ladder (G19): the 2H Q25 anchor - prism edges $3 : 4 : 5 : 12$ with volume 1125 gives $72k^3 = 1125$, $k = 2.5$, and total edge length $60k = 150$ cm.

Check yourself

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Notation and angle facts with named reasons; polygon sums derived; quadrilateral properties; constructions and loci with arcs kept; transformations incl. negative factors and invariance under combinations.

Congruence criteria and proof chains; circle parts and the eight theorems applied and proved; perimeters, areas, volumes incl. sectors, frustums and exact π ; plans, elevations and bearings; the $k : k^2 : k^3$ ladder.

Pythagoras and trig in 2D and 3D; exact trig values; sine and cosine rules; half $ab \sin C$; vectors in column form with collinearity proofs.

Exercises 2.1·2.3

2.1 A trapezium has parallel sides 6 cm and 10 cm with perpendicular height 4 cm. Find its area. [2]

2.2 A triangle has base 8 cm and perpendicular height 5 cm. Find its area. [2]

2.3 A circle has radius 4 cm. Write down its circumference and area in terms of π . [3]

Exercises 2.1·2.3

2.1 A trapezium has parallel sides 6 cm and 10 cm with perpendicular height 4 cm. Find its area. [2]

$$\mathbf{2.1} \quad \frac{1}{2}(6 + 10) \times 4 = 32 \text{ cm}^2.$$

2.2 A triangle has base 8 cm and perpendicular height 5 cm. Find its area. [2]

$$\mathbf{2.2} \quad \frac{1}{2} \times 8 \times 5 = 20 \text{ cm}^2.$$

2.3 A circle has radius 4 cm. Write down its circumference and area in terms of π . [3]

$$\mathbf{2.3} \quad C = 2\pi(4) = 8\pi \text{ cm}; A = \pi(4)^2 = 16\pi \text{ cm}^2.$$

Exercises 2.4·2.6

2.4 A cylinder has radius 3 cm and height 5 cm. Find its volume in terms of π . [3]

2.5 Find the perimeter of a semicircle of radius 4 cm, in terms of π . [3]

2.6 A cone has radius 3 cm and perpendicular height 4 cm. Find its slant height and its volume in terms of π . [2]

Exercises 2.4·2.6

2.4 A cylinder has radius 3 cm and height 5 cm. Find its volume in terms of π . [3]

2.4 $\pi \times 3^2 \times 5 = 45\pi \text{ cm}^3$.

2.5 Find the perimeter of a semicircle of radius 4 cm, in terms of π . [3]

2.5 Half circumference $= 4\pi$ plus diameter 8: perimeter $= 4\pi + 8 \text{ cm}$.

2.6 A cone has radius 3 cm and perpendicular height 4 cm. Find its slant height and its volume in terms of π . [2]

2.6 Slant $= \sqrt{3^2 + 4^2} = 5 \text{ cm}$; volume $= \frac{1}{3}\pi(3)^2(4) = 12\pi \text{ cm}^3$.

Exercises 3.1·3.2

3.1 (*June 2024 1H Q12, adapted.*) A sector of a circle has radius 12 cm and angle 60° . Work out the length of its arc, giving your answer in terms of π . [3]

3.2 (*Original, Higher similarity.*) Two spheres have radii 3 cm and 6 cm. (a) Write down the ratio of their surface areas. (b) The smaller sphere has volume $36\pi \text{ cm}^3$. Find the volume of the larger sphere. [4]

Exercises 3.1·3.2

3.1 (*June 2024 1H Q12, adapted.*) A sector of a circle has radius 12 cm and angle 60° . Work out the length of its arc, giving your answer in terms of π . [3]

$$\mathbf{3.1} \text{ Arc} = \frac{60}{360} \times 2 \times 12 \times \pi = \frac{1}{6} \times 24\pi = 4\pi \text{ cm.}$$

3.2 (*Original, Higher similarity.*) Two spheres have radii 3 cm and 6 cm. (a) Write down the ratio of their surface areas. (b) The smaller sphere has volume $36\pi \text{ cm}^3$. Find the volume of the larger sphere. [4]

3.2 (a) Length factor $k = 2$, so surface areas scale by k^2 : ratio $9 : 36 = 1 : 4$. (b) Volumes scale by $k^3 = 8$: $36\pi \times 8 = 288\pi \text{ cm}^3$.

Exercise 3.3

3.3 (*June 2024 2H Q25, adapted.*) A right-angled triangular prism has edges in the ratio $a : b : c : d = 3 : 4 : 5 : 12$ and volume 1125 cm^3 . Work out the total length of all of its edges. [5]

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3.3 Write the edges as $3k, 4k, 5k, 12k$: volume $= \frac{1}{2}(3k)(4k)(12k) = 72k^3 = 1125$, so $k^3 = 15.625$ and $k = 2.5$. Total edge length $= 2(3 + 4 + 5)k + 3 \times 12k = 60k = 150 \text{ cm}$ – two of each cross-section edge plus three length edges.

Local review guidance: the $\frac{\theta}{360}$ fraction or the $k : k^2 : k^3$ ladder stated before multiplying; exact π forms where asked, squared or cubed units correct; $72k^3 = 1125$ then cube-rooted. This is authored classroom guidance.