

Exercise walk-through

4.1.99

Independent transfer: Foundation then Higher

AQA · GCSE · Mathematics

67. Angle facts and polygon reasoning · Foundation

Angle facts and polygon reasoning · Foundation

If a straight-line angle is 68° , its neighbour is 112° ; its vertically opposite angle is 68° . A parallel-line corresponding angle is also 68° , with the reason named. Drawing a line through a triangle vertex parallel to the opposite side transfers its two base angles by alternate-angle equality; the three angles then form a straight line and sum to 180° . A pentagon splits into three triangles, giving 540° . A regular hexagon has total 720° , each interior angle 120° and each exterior turn 60° .

74. Triangle congruence and geometric proof · Foundation

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For an isosceles triangle ABC with $AB=AC$, let D be the midpoint of BC . Triangles ABD and ACD have $AB=AC$, $BD=DC$ and shared AD , so SSS gives congruence. Matching base angles ABC and BCA are therefore equal; the two angles at D are equal and form 180° , so each is 90° . To derive Pythagoras, arrange four congruent right triangles with legs a, b around a tilted square of side c inside a square of side $a+b$. Area gives $(a+b)^2 = 4(ab/2) + c^2$, hence $a^2 + b^2 = c^2$. For $a=3, b=4$, $c^2 = 9 + 16 = 25$, so $c=5$. These arguments establish results independently of a scale drawing.

176. Vector geometry and midpoint arguments · Higher

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Let $OA=a$ and $OB=b$, with M midpoint of OA and N midpoint of OB . Then $OM=a/2$ and $ON=b/2$, so $MN=b/2-a/2=(b-a)/2=AB/2$. Thus MN is parallel to AB and half as long. For $a=(4,2), b=(2,6)$, $M=(2,1), N=(1,3)$, $AB=(-2,4)$ and $MN=(-1,2)$, verifying the general result. In parallelogram $OACB$ with $OC=a+b$, the midpoint of OC is $(a+b)/2$; the midpoint of AB is the same, so the diagonals bisect each other. If $AP=3AB/2$, P lies on line AB beyond B ; if $AP=AB/2$, P is its midpoint. A parallel vector at another location alone does not prove three specified points collinear.

Exercise 2.1

2.1 Foundation: A cylindrical container has internal radius 3 cm and height 10 cm. Find its capacity in cubic centimetres and litres. A rectangular face is 6 cm by 8 cm; find its diagonal, naming the condition for your method.

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2.1 Cylinder volume $V = \pi r^2 h = \pi(3 \text{ cm})^2(10 \text{ cm}) = 90\pi \text{ cm}^3$. Since 1000 cubic centimetres is one litre, capacity is $0.09\pi \text{ L}$, about 0.283 L. A rectangular face has perpendicular sides. Pythagoras gives $d = \sqrt{a^2 + b^2} = \sqrt{(6 \text{ cm})^2 + (8 \text{ cm})^2} = 10 \text{ cm}$. A diagonal is not an extra edge of the rectangle's perimeter.

Exercise 3.1

3.1 Higher: In a circle, AB is a diameter and C lies elsewhere on the circumference. Angle CAB is 32° . Find angles ACB and ABC . Find the acute angle between the tangent at A and AC , explaining the circle theorem and which side of the tangent you mean.

Exercise 3.1

3.1 Higher: In a circle, AB is a diameter and C lies elsewhere on the circumference. Angle CAB is 32 degrees. Find angles ACB and ABC. Find the acute angle between the tangent at A and AC, explaining the circle theorem and which side of the tangent you mean.

3.1 Angle ACB is 90 degrees because an angle in a semicircle is a right angle. Triangle angles give $\angle ABC = 180 - 90 - 32 = 58$ degrees. By the alternate segment theorem, the angle between chord AC and the appropriate tangent ray equals angle ABC, 58 degrees. The other angle on the tangent's straight line is 122 degrees. Naming the acute angle selects 58; the drawing's apparent scale is unnecessary. The diameter and tangent facts are required hypotheses.

Local guidance: select and explain the relationship; calculate every requested result; check units, scope, conditions and completeness. These are authored classroom criteria.