

Exercise walk-through

3.3

Compound measures and rates of change (R6–R8, R13–R14)

AQA · GCSE · Mathematics

2. Units, ratio and sharing

Units, ratio and sharing

Change freely between standard units; compound units divide: speed = distance/time, density = mass/volume, pressure = force/area - algebraic at Higher: $V = 2x^3$ and $m = 5x^2$ give density $5/(2x)$.

Ratio reduces to simplest form ($45 : 60 = 3 : 4$); share by parts/total (350 pounds in $2 : 5$ gives 100 and 250); express one quantity as a fraction of another; best-buy problems compare unit prices.

5. Rates and compound units

Rates and compound units

Compound units - speed, rates of pay, unit pricing, density, pressure - including comparisons: 234 km in 2 h 36 min averages $234 / 2.6 = 90$ km/h; 9.20 per hour for 7.5 hours pays 69.

Interpret the gradient of a straight-line graph as a rate of change: distance-time gradient is speed; velocity-time gradient is acceleration and its area is distance.

Check yourself

Check yourself

Unit and compound-unit conversions incl. density and pressure (algebraic at Higher); ratio notation, sharing and best-buy contexts; percentage multipliers incl. original value and simple interest.

Direct and inverse proportion graphically and algebraically with constructed equations; the $k : k^2 : k^3$ ladder; gradients as average (chord) and instantaneous (tangent) rates; compound growth and decay with iterative processes.

Exercises 2.1·2.3

2.1 $y \propto x$ with $y = 21$ at $x = 7$: find the equation and y at $x = 12$ [3]

2.2 $y \propto x^2$ with $y = 48$ at $x = 4$: find y at $x = 7$ [3]

2.3 p is inversely proportional to x ; $p = 8$ when $x = 6$. Find p when $x = 8$. [3]

Exercises 2.1·2.3

2.1 $y \propto x$ with $y = 21$ at $x = 7$: find the equation and y at $x = 12$ [3]

2.1 $k = 3$, so $y = 3x$; $y = 36$.

2.2 $y \propto x^2$ with $y = 48$ at $x = 4$: find y at $x = 7$ [3]

2.2 $k = 3$; $y = 3 \times 49 = 147$.

2.3 p is inversely proportional to x ; $p = 8$ when $x = 6$. Find p when $x = 8$. [3]

2.3 $k = 48$; $p = 48 \div 8 = 6$.

Exercises 2.4·3.1

2.4 A job pays £9.20 per hour. Find the pay for 7.5 hours. [2]

2.5 On a distance-time graph a line rises 90 units of distance per 1.3 units of time. What rate does the gradient give, and what is its value to 3 s.f.? [3]

3.1 (*Original, Higher construct.*) y is directly proportional to the square of x . When $x = 3$, $y = 36$. (a) Find the equation linking x and y . (b) Find y when $x = 5$. (c) Find x when $y = 100$. [5]

Exercises 2.4·3.1

2.4 A job pays £9.20 per hour. Find the pay for 7.5 hours. [2]

2.4 $9.2 \times 7.5 = £69$.

2.5 On a distance-time graph a line rises 90 units of distance per 1.3 units of time. What rate does the gradient give, and what is its value to 3 s.f.? [3]

2.5 The gradient is the speed: $90 \div 1.3 = 69.2$ (3 s.f.).

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3.1 (a) $36 = 9k$, so $k = 4$ and $y = 4x^2$. (b) $y = 100$. (c) $4x^2 = 100$ gives $x = 5$.

Exercises 3.2·3.3

3.2 (*Original, Higher inverse construction.*) T is inversely proportional to the square root of m . $T = 6$ when $m = 4$. (a) Find the equation. (b) Find T when $m = 36$. [4]

3.3 (*Original, Higher tangent rate.*) A curve passes through the point where $x = 2$; the tangent there rises 5 units of y per 0.4 units of x . State the instantaneous rate of change at $x = 2$ and explain how it differs from the average rate over an interval. [3]

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3.2 (a) $T = \frac{k}{\sqrt{m}}$: $6 = \frac{k}{2}$, so $k = 12$ and $T = \frac{12}{\sqrt{m}}$. (b) $T = \frac{12}{6} = 2$.

3.3 (*Original, Higher tangent rate.*) A curve passes through the point where $x = 2$; the tangent there rises 5 units of y per 0.4 units of x . State the instantaneous rate of change at $x = 2$ and explain how it differs from the average rate over an interval. [3]

3.3 Rate = $5 \div 0.4 = 12.5$. The tangent gives the INSTANTANEOUS rate at the single point; the average rate uses a chord joining the interval's end points and can differ on a curve.

Local review guidance: k found from one pair before use; exact values; tangent vs chord distinction stated in words. This is authored classroom guidance.