

Exercise walk-through

# 2.5

## Quadratics and simultaneous equations - HT (A17–A23)

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AQA · GCSE · Mathematics

# 1. Unit 3.2 at a glance

## Unit 3.2 at a glance

Foundation rows A1-A25 across three tiers (basic foundation, additional foundation, Higher only). Higher-only additions: surds and algebraic fractions, identities and proofs, inverse and composite functions, perpendicular lines, turning points by completing the square, cubic/reciprocal/exponential/trig graphs, the circle equation with tangents, the quadratic formula, linear-quadratic simultaneity, iteration, set-notation inequality regions, quadratic and surd-ratio sequences.

Must-know (Formulae Sheet gives only the quadratic formula):  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ ;  $a(x + b)^2 + c$  reads the turning point; perpendicular  $m_1 m_2 = -1$ ;  $x_{[n+1]} = f(x_n)$  with suffix notation; quadratic nth term with half the second difference.

## 2. Notation and manipulation

### Notation and manipulation

$ab$  for  $a \times b$ ,  $3y$ ,  $a^2$ ,  $a/b$ ; coefficients as fractions not decimals; answers in simplest form. Collect, bracket, factorise, index laws; two binomials and  $x^2 + bx + c$  (additional); surds and algebraic fractions (Higher):  $(3 + \sqrt{2})(1 - 2\sqrt{2}) = -1 - 5\sqrt{2}$ .

Identities hold for ALL values -  $(x+2)^2 = x^2 + 4x + 4$ , and the 3H June 2024 verdict is exactly 'true for all values of  $x$ '. Proofs expand and compare; examples never prove. Rearrange formulae to change the subject:  $v = u + at$  gives  $t = (v - u)/a$ .

## 5. Solving

### Solving

Linear with the unknown both sides:  $7x - 22 = 4x + 29$  gives  $3x = 51$ ,  $x = 17$  (the 1H June 2024 anchor). Quadratics by factorising, formula, or completing the square:  $x^2 + 6x + 2 = 0$  gives  $x = -3 \pm \sqrt{7}$ . Simultaneous pairs keep both roots.

Iteration (Higher): from  $x^3 - 3x - 1 = 0$ ,  $x_{[n+1]} = (3x_n + 1)^{1/3}$  from  $x_1 = 1$  gives 1.587, 1.793, 1.855, 1.872, converging to 1.879 (3 dp) - check back in the original. Inequalities: number line (open strict, closed included), set notation at Higher:  $x^2 - x - 6 > 0$  gives  $\{x: x < -2\}$  union  $\{x: x > 3\}$ ; dashed boundaries for strict regions.

## Exercises 2.1·2.3

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**2.1** Write the next two terms of 5, 9, 17, 29, 45, ... [2]

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**2.2** Find the  $n$ th term of 4, 7, 12, 19, 28, ... [3]

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**2.3** Find the  $n$ th term of 6, 11, 18, 27, 38, ... [3]

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**2.1** Write the next two terms of 5, 9, 17, 29, 45, ... [2]

**2.1** Differences 20, 24 follow the +4 pattern: terms 65, 89.

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**2.2** Find the  $n$ th term of 4, 7, 12, 19, 28, ... [3]

**2.2** Differences 3, 5, 7, 9; second difference 2, so  $a = 1$ :  $n^2$  leaves 3, 3, 3, 3, giving  $n^2 + 3$ .

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**2.3** Find the  $n$ th term of 6, 11, 18, 27, 38, ... [3]

**2.3** Differences 5, 7, 9, 11; second difference 2, so  $a = 1$ : subtracting  $n^2$  (1, 4, 9, 16) leaves 5, 7, 9, 11 =  $2n + 3$ :  $n^2 + 2n + 3$ .

## Exercises 2.4·3.1

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**2.4** A Fibonacci-type sequence starts  $2, 5, \dots$ : write the next three terms. [2]

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**2.5** Which term of the sequence with  $n$ th term  $3n + 2$  equals 62? [2]

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**3.1** (*Original.*) A sequence has  $n$ th term  $n^2 + 2n$ . (a) Write down the first four terms.  
(b) Which term equals 168? [4]

## Exercises 2.4·3.1

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**2.4**  $7, 12, 19$ .

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**2.5** Which term of the sequence with  $n$ th term  $3n + 2$  equals 62? [2]

**2.5**  $3n + 2 = 62$  gives  $n = 20$ .

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**3.1** (*Original.*) A sequence has  $n$ th term  $n^2 + 2n$ . (a) Write down the first four terms.  
(b) Which term equals 168? [4]

**3.1** (a)  $3, 8, 15, 24$ . (b)  $n^2 + 2n = 168$  gives  $(n + 1)^2 = 169$ , so  $n = 12$ .

## Exercises 3.2·3.3

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**3.2** (*Original.*) The first differences of a sequence are 5, 9, 13, 17, .... (a) Find the next difference. (b) The first term is 3: write the first four terms. (c) Find the  $n$ th term. [5]

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**3.3** (*Original.*)  $y$  is proportional to  $x^2$ ;  $y = 48$  when  $x = 4$ . (a) Find the equation. (b) Find  $y$  when  $x = 7$ . [4]

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**3.2** (a) Differences increase by 4: next is 21. (b) 3, 8, 17, 30. (c) Second difference of the terms is 4, so  $a = 2$ ; subtracting  $2n^2$  (which gives 2, 8, 18, 32) leaves 1, 0,  $-1$ ,  $-2 = 2 - n$ : so the  $n$ th term is  $2n^2 - n + 2$  – check  $n = 4$ :  $32 - 4 + 2 = 30$ . ✓

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**3.3** (a)  $y = kx^2$ :  $48 = 16k$  gives  $k = 3$ . (b)  $y = 3 \times 49 = 147$ .

Local review guidance: second differences before guessing  $a$ ;  $n$ th terms checked on two positions; sequence types named. This is authored classroom guidance.