

Exercise walk-through

2.5.3

Equation Iteration

AQA · GCSE · Mathematics

21. Quadratics and inequalities · Higher

Quadratics and inequalities · Higher

The equation $x^2 - 10x + 21 = 0$ factorises as $(x-3)(x-7) = 0$, giving roots 3 and 7. Complete the square: $x^2 - 10x + 21 = (x-5)^2 - 4$, giving turning point $(5, -4)$ and symmetry line $x = 5$. The formula $x = [-b \pm \sqrt{(b^2 - 4ac)}] / (2a)$ also gives $(10 \pm 4) / 2 = 3, 7$. For $2x^2 + 5x - 3 = 0$, the discriminant is 49 and roots are $(-5 \pm 7) / 4 = 1/2, -3$. A graph gives approximate roots when exact factorisation is inconvenient. For an enclosure with area $A = x(10 - x)$, complete the square to get $A = 25 - (x-5)^2$. The greatest area is 25 at $x = 5$, within $0 < x < 10$.

153. Numerical iteration for equation roots · Higher

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For $x^2 - x - 2 = 0$, use $x_{\text{next}} = \sqrt{x+2}$ with $x_0 = 1$: $x_1 = \sqrt{3} \approx 1.73205$, $x_2 \approx 1.93185$, $x_3 \approx 1.98289$. The positive fixed point is 2 because $2 = \sqrt{4}$ and $2^2 - 2 - 2 = 0$. The rearrangement only seeks a nonnegative root; the original equation also has root -1. Using $x_{\text{next}} = x^2 - 2$ from 3 instead gives 7 then 47, so a different rearrangement can diverge.

191. Reasoning and check

Reasoning and check

The equation excludes $x=3$. Cancellation gives $x+3 = x^2 - 5$, hence $x^2 - x - 8 = 0$ and $x = (1 \pm \sqrt{33})/2$. Neither is 3, so both remain. For the separate inequality, $(x-5)(x+1) \leq 0$ gives $-1 \leq x \leq 5$, including both zero endpoints. Completing the square gives $y = (x-2)^2 - 9$; its turning point is $(2,-9)$, a minimum, with roots -1 and 5. Do not confuse the two different quadratics.

Exercises 2.1·2.3

2.1 (Higher) Find x_1 from $x_{\text{next}} = \sqrt{x+2}$, $x_0 = 1$.

2.2 (Higher) Find the positive fixed point.

2.3 (Higher) Find the original-equation residual at $x=2$.

3 Exam-style questions

Exercises 2.1·2.3

2.1 (Higher) Find x_1 from $x_{\text{next}} = \sqrt{x+2}$, $x_0 = 1$.

2.1 $\sqrt{1+2} = \sqrt{3}$.

2.2 (Higher) Find the positive fixed point.

2.2 Solve $L^2 - L - 2 = 0$ and retain the nonnegative solution.

2.3 (Higher) Find the original-equation residual at $x = 2$.

3 Exam-style questions

2.3 $2^2 - 2 - 2 = 0$.

Exercise 3.1

3.1 Decide whether this claim is true. Justify with a concrete calculation or counterexample: Every rearrangement of an equation converges from every starting value.

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3.1 False. The equation $x=2$ can be rewritten $x=2x-2$. Iteration $x_{\text{next}}=2x-2$ from $x_0=3$ gives 4,6,10,... and diverges, although 2 is a fixed point.

Local review guidance: show the chosen method; give the checked result; explain the condition or failed shortcut. This is authored classroom guidance.