

Exercise walk-through

2.5.1

Quadratic

AQA · GCSE · Mathematics

21. Quadratics and inequalities · Higher

Quadratics and inequalities · Higher

The equation $x^2 - 10x + 21 = 0$ factorises as $(x-3)(x-7) = 0$, giving roots 3 and 7. Complete the square: $x^2 - 10x + 21 = (x-5)^2 - 4$, giving turning point $(5, -4)$ and symmetry line $x = 5$. The formula $x = [-b \pm \sqrt{(b^2 - 4ac)}] / (2a)$ also gives $(10 \pm 4) / 2 = 3, 7$. For $2x^2 + 5x - 3 = 0$, the discriminant is 49 and roots are $(-5 \pm 7) / 4 = 1/2, -3$. A graph gives approximate roots when exact factorisation is inconvenient. For an enclosure with area $A = x(10 - x)$, complete the square to get $A = 25 - (x-5)^2$. The greatest area is 25 at $x = 5$, within $0 < x < 10$.

183. Reasoning and check

Reasoning and check

For $F(x) = (x^2 - 9)/(x - 3)$, factor $x^2 - 9 = (x - 3)(x + 3)$. Thus $F(x) = x + 3$ for $x \neq 3$; $F(5) = 8$, but $F(3)$ is undefined, not 6. For addition, $1/(x - 1) + 2/(x + 1) = [(x + 1) + 2(x - 1)]/[(x - 1)(x + 1)] = (3x - 1)/(x^2 - 1)$, with $x \neq \pm 1$. At $x = 3$, both forms give 1. For subtraction, the numerator is $(x + 1) - 2(x - 1) = 3 - x$, so the minus sign acts on both terms. Multiplication $[(x - 1)/(x + 2)] \times [(x + 2)/(x + 1)]$ gives $(x - 1)/(x + 1)$, but the original excludes $x = -2$ and $x = -1$. Division $[(x^2 - 1)/(x^2 + 3x + 2)] \div [(x - 1)/(x + 2)]$ gives 1, yet $x = -2, -1, 1$ are excluded: $x = 1$ makes the divisor zero. Finally $F(x) = 6$ reduces to $x + 3 = 6$.

191. Reasoning and check

Reasoning and check

The equation excludes $x=3$. Cancellation gives $x+3 = x^2 - 5$, hence $x^2 - x - 8 = 0$ and $x = (1 \pm \sqrt{33})/2$. Neither is 3, so both remain. For the separate inequality, $(x-5)(x+1) \leq 0$ gives $-1 \leq x \leq 5$, including both zero endpoints. Completing the square gives $y = (x-2)^2 - 9$; its turning point is $(2,-9)$, a minimum, with roots -1 and 5. Do not confuse the two different quadratics.

Exercises 2.1·2.3

2.1 (Foundation) Find the smaller root of $x^2-10x+21=0$.

2.2 (Foundation) Find the larger root of $x^2-10x+21=0$.

2.3 (Foundation) Find $(5-3)(5-7)$.

Exercises 2.1·2.3

2.1 (Foundation) Find the smaller root of $x^2-10x+21=0$.

2.1 Factorise to $(x-3)(x-7)=0$. The smaller root is 3.

2.2 (Foundation) Find the larger root of $x^2-10x+21=0$.

2.2 The factorised roots are 3 and 7; the larger is 7.

2.3 (Foundation) Find $(5-3)(5-7)$.

2.3 Substitute 5: $(5-3)(5-7)=2 \times (-2)=-4$.

Exercises 2.4·3.1

2.4 (Higher) Find the discriminant of $x^2-6x+9=0$.

2.5 (Higher) Find the maximum value of $x(10-x)$.

3 Exam-style questions

3.1 Decide whether this claim is true. Justify with a concrete calculation or counterexample: A positive discriminant means that a quadratic has no real roots.

Exercises 2.4·3.1

2.4 (Higher) Find the discriminant of $x^2-6x+9=0$.

2.4 The discriminant is $(-6)^2-4\times 1\times 9=0$.

2.5 (Higher) Find the maximum value of $x(10-x)$.

3 Exam-style questions

2.5 Complete the square: $x(10-x)=25-(x-5)^2$. The maximum is 25.

3.1 Decide whether this claim is true. Justify with a concrete calculation or counterexample: A positive discriminant means that a quadratic has no real roots.

3.1 False. $x^2-1=0$ has discriminant $4>0$ and two distinct real roots, -1 and 1 . A negative discriminant means no real roots.

Local review guidance: show the chosen method; give the checked result; explain the condition or failed shortcut. This is authored classroom guidance.