

Exercise walk-through

2.4

Graphs: linear, quadratic, reciprocal, real contexts (A8–A16)

AQA · GCSE · Mathematics

1. Unit 3.2 at a glance

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Foundation rows A1-A25 across three tiers (basic foundation, additional foundation, Higher only). Higher-only additions: surds and algebraic fractions, identities and proofs, inverse and composite functions, perpendicular lines, turning points by completing the square, cubic/reciprocal/exponential/trig graphs, the circle equation with tangents, the quadratic formula, linear-quadratic simultaneity, iteration, set-notation inequality regions, quadratic and surd-ratio sequences.

Must-know (Formulae Sheet gives only the quadratic formula): $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$; $a(x + b)^2 + c$ reads the turning point; perpendicular $m_1 m_2 = -1$; $x_{[n+1]} = f(x_n)$ with suffix notation; quadratic nth term with half the second difference.

4. Graphs: lines to curves

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Gradient $m = (y_2 - y_1)/(x_2 - x_1)$; perpendicular $m_1 m_2 = -1$; through $(1, 7)$ and $(2, 9)$: $y = 2x + 5$, perpendicular through $(1, 7)$: $y = -1/2(x - 1) + 7$. Completing the square: $x^2 - 6x + 5 = (x - 3)^2 - 4$, minimum $(3, -4)$, symmetry $x = 3$.

Sketch families: linear, quadratic, cubic, reciprocal $1/x$, exponential k^x , and sin/cos/tan for any angle (Higher); translations $f(x) + a$ up, $f(x + a)$ left.

Real-context and kinematic graphs: velocity-time area is distance. Circle $x^2 + y^2 = r^2$; tangent gradient at (x_1, y_1) is $-x_1/y_1$.

5. Solving

Solving

Linear with the unknown both sides: $7x - 22 = 4x + 29$ gives $3x = 51$, $x = 17$ (the 1H June 2024 anchor). Quadratics by factorising, formula, or completing the square: $x^2 + 6x + 2 = 0$ gives $x = -3 \pm \sqrt{7}$. Simultaneous pairs keep both roots.

Iteration (Higher): from $x^3 - 3x - 1 = 0$, $x_{[n+1]} = (3x_n + 1)^{1/3}$ from $x_1 = 1$ gives 1.587, 1.793, 1.855, 1.872, converging to 1.879 (3 dp) - check back in the original. Inequalities: number line (open strict, closed included), set notation at Higher: $x^2 - x - 6 > 0$ gives $\{x: x < -2\}$ union $\{x: x > 3\}$; dashed boundaries for strict regions.

Exercises 2.1·2.3

2.1 Solve $3x - 2 < 10$ and represent on a number line [2]

2.2 Solve $7 - x \leq 5$ [2]

2.3 Solve $-7 \leq 3x + 2 < 6$ [3]

Exercises 2.1·2.3

2.1 Solve $3x - 2 < 10$ and represent on a number line [2]

2.1 $x < 4$: open circle at 4, arrow left.

2.2 Solve $7 - x \leq 5$ [2]

2.2 $-x \leq -2$, so (flip) $x \geq 2$.

2.3 Solve $-7 \leq 3x + 2 < 6$ [3]

2.3 $-9 \leq 3x < 4$, so $-3 \leq x < \frac{4}{3}$.

Exercises 2.4·3.1

2.4 Solve $x^2 - x - 6 > 0$ [3]

2.5 With $x_{n+1} = \frac{1}{2}x_n + 3$ from $x_1 = 1$, find x_2 , x_3 , x_4 and state the limit. [3]

3.1 (*Original.*) The equation $x^3 - 3x - 1 = 0$ has a root between 1 and 2. (a) Show that rearranging gives $x = \sqrt[3]{3x+1}$. (b) Use the iteration $x_{n+1} = \sqrt[3]{3x_n+1}$ from $x_1 = 1$ to find x_4 . (c) Verify the root is 1.879 to 3 d.p. [5]

Exercises 2.4·3.1

2.4 Solve $x^2 - x - 6 > 0$ [3]

2.4 $(x - 3)(x + 2) > 0$: positive outside the roots, so $x < -2$ or $x > 3$.

2.5 With $x_{n+1} = \frac{1}{2}x_n + 3$ from $x_1 = 1$, find x_2, x_3, x_4 and state the limit. [3]

2.5 $x_2 = 3.5, x_3 = 4.75, x_4 = 5.375$; limit $x = 6$ (solves $x = \frac{1}{2}x + 3$).

3.1 (*Original.*) The equation $x^3 - 3x - 1 = 0$ has a root between 1 and 2. (a) Show that rearranging gives $x = \sqrt[3]{3x + 1}$. (b) Use the iteration $x_{n+1} = \sqrt[3]{3x_n + 1}$ from $x_1 = 1$ to find x_4 . (c) Verify the root is 1.879 to 3 d.p. [5]

3.1 (a) $x^3 = 3x + 1$ from the equation, so $x = \sqrt[3]{3x + 1}$. (b) $x_2 = 1.587, x_3 = 1.793, x_4 = 1.855$. (c) Continuing: $x_5 = 1.872, x_6 = 1.877$; and $1.8785^3 - 3(1.8785) - 1 < 0 < 1.8795^3 - 3(1.8795) - 1$, so the root rounds to 1.879.

Exercises 3.2·3.3

3.2 (*Original.*) Solve $x^2 + 3x + 2 > 0$ and write the solution set in set notation. [4]

3.3 (*Original.*) Shade or describe the region satisfying $x \geq 0$, $y \geq 1$ and $x + y \leq 5$, stating the vertices. [3]

Exercises 3.2·3.3

3.2 (*Original.*) Solve $x^2 + 3x + 2 > 0$ and write the solution set in set notation. [4]

3.2 $(x + 1)(x + 2) > 0$: $x < -2$ or $x > -1$, i.e. $\{x: x < -2\} \cup \{x: x > -1\}$.

3.3 (*Original.*) Shade or describe the region satisfying $x \geq 0$, $y \geq 1$ and $x + y \leq 5$, stating the vertices. [3]

3.3 Triangle with vertices $(0, 1)$, $(0, 5)$, $(4, 1)$; shade inside.

Local review guidance: rearrangement shown before iterating; iterations to the stated accuracy; inequality notation matches strict/included endpoints. This is authored classroom guidance.