

Exercise walk-through

2.4.3

Functions

AQA · GCSE · Mathematics

21. Quadratics and inequalities · Higher

Quadratics and inequalities · Higher

The equation $x^2 - 10x + 21 = 0$ factorises as $(x-3)(x-7) = 0$, giving roots 3 and 7. Complete the square: $x^2 - 10x + 21 = (x-5)^2 - 4$, giving turning point $(5, -4)$ and symmetry line $x = 5$. The formula $x = [-b \pm \sqrt{(b^2 - 4ac)}] / (2a)$ also gives $(10 \pm 4) / 2 = 3, 7$. For $2x^2 + 5x - 3 = 0$, the discriminant is 49 and roots are $(-5 \pm 7) / 4 = 1/2, -3$. A graph gives approximate roots when exact factorisation is inconvenient. For an enclosure with area $A = x(10 - x)$, complete the square to get $A = 25 - (x-5)^2$. The greatest area is 25 at $x = 5$, within $0 < x < 10$.

141. Graphical gradients and area estimates · Higher

Graphical gradients and area estimates · Higher

For a distance curve $d=t^2$, the chord from (1,1) to (3,9) has gradient $(9-1)/(3-1)=4$. A tangent at (2,4) passes through (1,0) and (3,8), giving gradient 4 without calculus. A velocity-time trapezium with endpoint velocities 2 and 6 m/s over 3 s has area $(2+6) \times 3/2 = 12$ m. Smaller strips can improve a curved-area estimate; curvature affects over/underestimation. A cost-versus-quantity tangent through (2,12) and (6,28) gives a local rate $(28-12)/(6-2)=4$ yuan per extra item, linking graph slope to a financial interpretation.

183. Reasoning and check

Reasoning and check

For $F(x) = (x^2 - 9)/(x - 3)$, factor $x^2 - 9 = (x - 3)(x + 3)$. Thus $F(x) = x + 3$ for $x \neq 3$; $F(5) = 8$, but $F(3)$ is undefined, not 6. For addition, $1/(x - 1) + 2/(x + 1) = [(x + 1) + 2(x - 1)]/[(x - 1)(x + 1)] = (3x - 1)/(x^2 - 1)$, with $x \neq \pm 1$. At $x = 3$, both forms give 1. For subtraction, the numerator is $(x + 1) - 2(x - 1) = 3 - x$, so the minus sign acts on both terms. Multiplication $[(x - 1)/(x + 2)] \times [(x + 2)/(x + 1)]$ gives $(x - 1)/(x + 1)$, but the original excludes $x = -2$ and $x = -1$. Division $[(x^2 - 1)/(x^2 + 3x + 2)] \div [(x - 1)/(x + 2)]$ gives 1, yet $x = -2, -1, 1$ are excluded: $x = 1$ makes the divisor zero. Finally $F(x) = 6$ reduces to $x + 3 = 6$.

Exercises 2.1·2.3

2.1 (Higher) For $f(x)=2x+3$, find $f^{-1}(11)$.

2.2 (Higher) For $f(x)=2x+3$ and $g(x)=x^2$, find $fg(4)$.

2.3 (Higher) For $f(x)=\sqrt{x-2}$, find $f(11)$.

3 Exam-style questions

Exercises 2.1·2.3

2.1 (Higher) For $f(x)=2x+3$, find f inverse(11).

2.1 Solve $2x+3=11$, giving $x=4$.

2.2 (Higher) For $f(x)=2x+3$ and $g(x)=x^2$, find $fg(4)$.

2.2 Apply g first: $g(4)=16$; then $f(16)=2 \times 16 + 3 = 35$.

2.3 (Higher) For $f(x)=\sqrt{(x-2)}$, find $f(11)$.

3 Exam-style questions

2.3 Substitute into the square root: $\sqrt{(11-2)}=\sqrt{9}=3$.

Exercise 3.1

3.1 Decide whether this claim is true. Justify with a concrete calculation or counterexample: Every quadratic function on all real numbers has an inverse function.

Exercise 3.1

3.1 Decide whether this claim is true. Justify with a concrete calculation or counterexample: Every quadratic function on all real numbers has an inverse function.

3.1 False. For $f(x)=x^2$, $f(-1)=f(1)=1$, so the inverse would have two outputs for input 1. Restricting the domain, for example $x \geq 0$, makes an inverse function possible.

Local review guidance: show the chosen method; give the checked result; explain the condition or failed shortcut. This is authored classroom guidance.