

Exercise walk-through

## 2.3

# Sequences, including $n$ th term (A24–A25)

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AQA · GCSE · Mathematics

# 1. Unit 3.2 at a glance

## Unit 3.2 at a glance

Foundation rows A1-A25 across three tiers (basic foundation, additional foundation, Higher only). Higher-only additions: surds and algebraic fractions, identities and proofs, inverse and composite functions, perpendicular lines, turning points by completing the square, cubic/reciprocal/exponential/trig graphs, the circle equation with tangents, the quadratic formula, linear-quadratic simultaneity, iteration, set-notation inequality regions, quadratic and surd-ratio sequences.

Must-know (Formulae Sheet gives only the quadratic formula):  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ ;  $a(x + b)^2 + c$  reads the turning point; perpendicular  $m_1 m_2 = -1$ ;  $x_{[n+1]} = f(x_n)$  with suffix notation; quadratic nth term with half the second difference.

## 6. Sequences

### Sequences

Term-to-term and position-to-term rules; triangular  $n(n+1)/2$ , squares, cubes, Fibonacci. Linear  $n$ th term  $an + b$ ; quadratic  $an^2 + bn + c$  with  $a =$  half the second difference.

For 5, 9, 17, 29, 45: differences 4, 8, 12, 16, second difference 4, so  $a = 2$  and the sequence is  $2n^2 - 2n + 5$  (check  $n = 4$ : 29). Geometric progressions with rational or surd ratios; recursive sequences defined in the question.

# Check yourself

## Check yourself

Notation and substitution; manipulation through surds and algebraic fractions; identities and proofs; formulae rearranged. Functions with inverse and composite.

Lines through points with perpendiculars; quadratic turning points by completing the square; the full graph-families bank with translations; circle equation and tangents; linear to linear-quadratic solving; iteration to a stated accuracy; inequalities with number lines, set notation and regions; sequence types and  $n$ th terms.

## Exercises 2.1·2.3

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**2.1** Solve  $5x - 3 = 2x + 9$  [2]

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**2.2** Solve  $3(x - 2) = 2(x + 5)$  [2]

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**2.3** Solve  $x^2 - 3x - 10 = 0$  [2]

## Exercises 2.1·2.3

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**2.1** Solve  $5x - 3 = 2x + 9$  [2]

**2.1**  $3x = 12$ , so  $x = 4$ .

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**2.2** Solve  $3(x - 2) = 2(x + 5)$  [2]

**2.2**  $3x - 6 = 2x + 10$ , so  $x = 16$ .

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**2.3** Solve  $x^2 - 3x - 10 = 0$  [2]

**2.3**  $(x - 5)(x + 2) = 0$ :  $x = 5$  or  $-2$ .

## Exercises 2.4·2.6

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**2.4** Solve  $x^2 + 6x + 2 = 0$ , giving exact roots [3]

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**2.5** Solve the pair  $y = 2x + 1$ ,  $y = x^2 - 2$  [4]

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**2.6** A rectangle's length exceeds its width by 5 m and its area is  $84 \text{ m}^2$ . Form and solve a quadratic for the dimensions. [4]

## Exercises 2.4·2.6

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**2.4** Solve  $x^2 + 6x + 2 = 0$ , giving exact roots [3]

**2.4**  $(x + 3)^2 = 7$ , so  $x = -3 \pm \sqrt{7}$ .

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**2.5** Solve the pair  $y = 2x + 1$ ,  $y = x^2 - 2$  [4]

**2.5**  $x^2 - 2 = 2x + 1$ , so  $x^2 - 2x - 3 = 0 = (x - 3)(x + 1)$ : points  $(3, 7)$  and  $(-1, -1)$ .

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**2.6** A rectangle's length exceeds its width by 5 m and its area is  $84 \text{ m}^2$ . Form and solve a quadratic for the dimensions. [4]

**2.6**  $w(w + 5) = 84$  gives  $w^2 + 5w - 84 = 0 = (w + 12)(w - 7)$ : width 7 m, length 12 m.

## Exercises 3.1·3.3

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**3.1** (*Edexcel-style, 1H June 2024 Q8 adapted.*) Solve  $7x - 22 = 4x + 29$ . [3]

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**3.2** (*Original.*) The line  $y = 2x + 5$  meets the curve  $y = x^2$ . (a) Show the  $x$ -coordinates satisfy  $x^2 - 2x - 5 = 0$ . (b) Find both intersection points, exact where irrational. [5]

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**3.3** (*Original.*) 120 tickets were sold for £340. Adult tickets cost £5 and child tickets £2.50. Form a pair of simultaneous equations and find each count. [4]

## Exercises 3.1·3.3

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**3.1** (*Edexcel-style, 1H June 2024 Q8 adapted.*) Solve  $7x - 22 = 4x + 29$ . [3]

**3.1**  $3x = 51$ , so  $x = 17$ .

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**3.2** (*Original.*) The line  $y = 2x + 5$  meets the curve  $y = x^2$ . (a) Show the  $x$ -coordinates satisfy  $x^2 - 2x - 5 = 0$ . (b) Find both intersection points, exact where irrational. [5]

**3.2** (a)  $x^2 = 2x + 5$ , so  $x^2 - 2x - 5 = 0$ . (b)  $x = 1 \pm \sqrt{6}$ ; the points are  $(1 + \sqrt{6}, 7 + 2\sqrt{6})$  and  $(1 - \sqrt{6}, 7 - 2\sqrt{6})$ .

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**3.3** (*Original.*) 120 tickets were sold for £340. Adult tickets cost £5 and child tickets £2.50. Form a pair of simultaneous equations and find each count. [4]

**3.3**  $a + c = 120$  and  $5a + 2.5c = 340$ : from the first  $a = 120 - c$ ;  $5(120 - c) + 2.5c = 340$  gives  $600 - 2.5c = 340$ , so  $c = 104$  and  $a = 16$ .

Local review guidance: substitution shown before expansion; BOTH solutions with both coordinates; context rejections justified. This is authored classroom guidance.