

Exercise walk-through

2.3.3

## Quadratic Sequences

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AQA · GCSE · Mathematics

## 21. Quadratics and inequalities · Higher

### Quadratics and inequalities · Higher

The equation  $x^2 - 10x + 21 = 0$  factorises as  $(x-3)(x-7) = 0$ , giving roots 3 and 7. Complete the square:  $x^2 - 10x + 21 = (x-5)^2 - 4$ , giving turning point  $(5, -4)$  and symmetry line  $x = 5$ . The formula  $x = [-b \pm \sqrt{(b^2 - 4ac)}] / (2a)$  also gives  $(10 \pm 4) / 2 = 3, 7$ . For  $2x^2 + 5x - 3 = 0$ , the discriminant is 49 and roots are  $(-5 \pm 7) / 4 = 1/2, -3$ . A graph gives approximate roots when exact factorisation is inconvenient. For an enclosure with area  $A = x(10 - x)$ , complete the square to get  $A = 25 - (x-5)^2$ . The greatest area is 25 at  $x = 5$ , within  $0 < x < 10$ .

## 183. Reasoning and check

### Reasoning and check

For  $F(x) = (x^2 - 9)/(x - 3)$ , factor  $x^2 - 9 = (x - 3)(x + 3)$ . Thus  $F(x) = x + 3$  for  $x \neq 3$ ;  $F(5) = 8$ , but  $F(3)$  is undefined, not 6. For addition,  $1/(x - 1) + 2/(x + 1) = [(x + 1) + 2(x - 1)]/[(x - 1)(x + 1)] = (3x - 1)/(x^2 - 1)$ , with  $x \neq \pm 1$ . At  $x = 3$ , both forms give 1. For subtraction, the numerator is  $(x + 1) - 2(x - 1) = 3 - x$ , so the minus sign acts on both terms. Multiplication  $[(x - 1)/(x + 2)] \times [(x + 2)/(x + 1)]$  gives  $(x - 1)/(x + 1)$ , but the original excludes  $x = -2$  and  $x = -1$ . Division  $[(x^2 - 1)/(x^2 + 3x + 2)] \div [(x - 1)/(x + 2)]$  gives 1, yet  $x = -2, -1, 1$  are excluded:  $x = 1$  makes the divisor zero. Finally  $F(x) = 6$  reduces to  $x + 3 = 6$ .

## 191. Reasoning and check

### Reasoning and check

The equation excludes  $x=3$ . Cancellation gives  $x+3 = x^2 - 5$ , hence  $x^2 - x - 8 = 0$  and  $x = (1 \pm \sqrt{33})/2$ . Neither is 3, so both remain. For the separate inequality,  $(x-5)(x+1) \leq 0$  gives  $-1 \leq x \leq 5$ , including both zero endpoints. Completing the square gives  $y = (x-2)^2 - 9$ ; its turning point is  $(2,-9)$ , a minimum, with roots -1 and 5. Do not confuse the two different quadratics.

## Exercises 2.1·2.3

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**2.1** (Higher) Find  $a$  when the second difference is 2.

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**2.2** (Higher) Find  $u_5$  for  $u_n = n^2 + 2n$ .

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**2.3** (Higher) Find the third term of the geometric sequence starting 1 with ratio  $\sqrt{2}$ .

## 3 Exam-style questions

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## Exercises 2.1·2.3

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**2.1** (Higher) Find  $a$  when the second difference is 2.

**2.1**  $2a=2$ , so  $a = 1$ .

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**2.2** (Higher) Find  $u_5$  for  $u_n=n^2+2n$ .

**2.2**  $25+10=35$ .

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**2.3** (Higher) Find the third term of the geometric sequence starting 1 with ratio  $\sqrt{2}$ .

## 3 Exam-style questions

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**2.3**  $1 \times \sqrt{2} \times \sqrt{2} = 2$ .

## Exercise 3.1

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**3.1** Decide whether this claim is true. Justify with a concrete calculation or counterexample: The coefficient of  $n^2$  always equals the second difference.

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**3.1** False. For  $n^2$  the terms 1,4,9 have first differences 3,5 and second difference 2. The coefficient is 1, half the constant second difference.

Local review guidance: show the chosen method; give the checked result; explain the condition or failed shortcut. This is authored classroom guidance.