

Exercise walk-through

2.2

Equations, inequalities, formulae and rearranging (A5–A7)

AQA · GCSE · Mathematics

1. Unit 3.2 at a glance

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Foundation rows A1-A25 across three tiers (basic foundation, additional foundation, Higher only). Higher-only additions: surds and algebraic fractions, identities and proofs, inverse and composite functions, perpendicular lines, turning points by completing the square, cubic/reciprocal/exponential/trig graphs, the circle equation with tangents, the quadratic formula, linear-quadratic simultaneity, iteration, set-notation inequality regions, quadratic and surd-ratio sequences.

Must-know (Formulae Sheet gives only the quadratic formula): $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$; $a(x + b)^2 + c$ reads the turning point; perpendicular $m_1 m_2 = -1$; $x_{[n+1]} = f(x_n)$ with suffix notation; quadratic nth term with half the second difference.

2. Notation and manipulation

Notation and manipulation

ab for $a \times b$, $3y$, a^2 , a/b ; coefficients as fractions not decimals; answers in simplest form. Collect, bracket, factorise, index laws; two binomials and $x^2 + bx + c$ (additional); surds and algebraic fractions (Higher): $(3 + \sqrt{2})(1 - 2\sqrt{2}) = -1 - 5\sqrt{2}$.

Identities hold for ALL values - $(x+2)^2 = x^2 + 4x + 4$, and the 3H June 2024 verdict is exactly 'true for all values of x '. Proofs expand and compare; examples never prove. Rearrange formulae to change the subject: $v = u + at$ gives $t = (v - u)/a$.

Check yourself

Check yourself

Notation and substitution; manipulation through surds and algebraic fractions; identities and proofs; formulae rearranged. Functions with inverse and composite.

Lines through points with perpendiculars; quadratic turning points by completing the square; the full graph-families bank with translations; circle equation and tangents; linear to linear-quadratic solving; iteration to a stated accuracy; inequalities with number lines, set notation and regions; sequence types and n th terms.

Exercises 2.1·2.3

2.1 For $f(x) = 3x - 5$ and $g(x) = x^2$, find $fg(2)$ [2]

2.2 Find $f^{-1}(x)$ for $f(x) = 3x - 5$ [2]

2.3 Find the equation of the line through $(1, 7)$ and $(2, 9)$ [3]

Exercises 2.1·2.3

2.1 For $f(x) = 3x - 5$ and $g(x) = x^2$, find $fg(2)$ [2]

2.1 $g(2) = 4$; $f(4) = 7$.

2.2 Find $f^{-1}(x)$ for $f(x) = 3x - 5$ [2]

2.2 $y = 3x - 5 \Rightarrow x = \frac{y+5}{3}$, so $f^{-1}(x) = \frac{x+5}{3}$.

2.3 Find the equation of the line through $(1, 7)$ and $(2, 9)$ [3]

2.3 $m = 2$, so $y = 2x + 5$.

Exercises 2.4·2.6

2.4 Write down the gradient of the line perpendicular to $m = \frac{2}{5}$ [1]

2.5 Write $y = x^2 - 6x + 5$ in the form $(x + a)^2 + b$ and state the minimum point [3]

2.6 The circle $x^2 + y^2 = 25$ passes through $(3, 4)$. Write down the gradient of the tangent there. [2]

Exercises 2.4·2.6

2.4 Write down the gradient of the line perpendicular to $m = \frac{2}{5}$ [1]

2.4 $-\frac{5}{2}$.

2.5 Write $y = x^2 - 6x + 5$ in the form $(x + a)^2 + b$ and state the minimum point [3]

2.5 $(x - 3)^2 - 4$; minimum $(3, -4)$.

2.6 The circle $x^2 + y^2 = 25$ passes through $(3, 4)$. Write down the gradient of the tangent there. [2]

2.6 Radius gradient $4/3$; tangent $-3/4$.

Exercises 3.1·3.3

3.1 (*Original.*) A line joins $A(-1, 3)$ to $B(3, 11)$. (a) Find its equation. (b) Find the equation of the perpendicular through the midpoint of AB . [5]

3.2 (*Original.*) Sketch or describe $y = 2^x$, giving the y -intercept and the behaviour as x grows and as $x \rightarrow -\infty$. [3]

3.3 (*Original.*) For $y = x^2 + 6x + 2$: (a) write in completed-square form; (b) hence solve $x^2 + 6x + 2 = 0$ giving exact roots. [4]

Exercises 3.1·3.3

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3.1 (a) $m = \frac{11-3}{3-(-1)} = 2$, so $y = 2x + 5$. (b) Midpoint $(1, 7)$; perpendicular gradient $-\frac{1}{2}$: $y - 7 = -\frac{1}{2}(x - 1)$, so $y = -\frac{1}{2}x + \frac{15}{2}$.

3.2 (*Original.*) Sketch or describe $y = 2^x$, giving the y -intercept and the behaviour as x grows and as $x \rightarrow -\infty$. [3]

3.2 Passes $(0, 1)$; rises ever faster as x grows; approaches the x -axis (never 0) as $x \rightarrow -\infty$.

3.3 (*Original.*) For $y = x^2 + 6x + 2$: (a) write in completed-square form; (b) hence solve $x^2 + 6x + 2 = 0$ giving exact roots. [4]

3.3 (a) $(x + 3)^2 - 7$. (b) $(x + 3)^2 = 7$, so $x = -3 \pm \sqrt{7}$.

Local review guidance: gradient then point-slope form; exact roots; graph descriptions name intercept AND end behaviour. This is authored classroom guidance.