

Exercise walk-through

# 2.2.1

## Formula Rearrangement

---

AQA · GCSE · Mathematics

## 21. Quadratics and inequalities · Higher

### Quadratics and inequalities · Higher

The equation  $x^2 - 10x + 21 = 0$  factorises as  $(x-3)(x-7) = 0$ , giving roots 3 and 7. Complete the square:  $x^2 - 10x + 21 = (x-5)^2 - 4$ , giving turning point  $(5, -4)$  and symmetry line  $x = 5$ . The formula  $x = [-b \pm \sqrt{(b^2 - 4ac)}] / (2a)$  also gives  $(10 \pm 4) / 2 = 3, 7$ . For  $2x^2 + 5x - 3 = 0$ , the discriminant is 49 and roots are  $(-5 \pm 7) / 4 = 1/2, -3$ . A graph gives approximate roots when exact factorisation is inconvenient. For an enclosure with area  $A = x(10 - x)$ , complete the square to get  $A = 25 - (x-5)^2$ . The greatest area is 25 at  $x = 5$ , within  $0 < x < 10$ .

## 153. Numerical iteration for equation roots · Higher

### Numerical iteration for equation roots · Higher

For  $x^2 - x - 2 = 0$ , use  $x_{\text{next}} = \sqrt{x+2}$  with  $x_0 = 1$ :  $x_1 = \sqrt{3} \approx 1.73205$ ,  $x_2 \approx 1.93185$ ,  $x_3 \approx 1.98289$ . The positive fixed point is 2 because  $2 = \sqrt{4}$  and  $2^2 - 2 - 2 = 0$ . The rearrangement only seeks a nonnegative root; the original equation also has root -1. Using  $x_{\text{next}} = x^2 - 2$  from 3 instead gives 7 then 47, so a different rearrangement can diverge.

## 183. Reasoning and check

### Reasoning and check

For  $F(x) = (x^2 - 9)/(x - 3)$ , factor  $x^2 - 9 = (x - 3)(x + 3)$ . Thus  $F(x) = x + 3$  for  $x \neq 3$ ;  $F(5) = 8$ , but  $F(3)$  is undefined, not 6. For addition,  $1/(x - 1) + 2/(x + 1) = [(x + 1) + 2(x - 1)]/[(x - 1)(x + 1)] = (3x - 1)/(x^2 - 1)$ , with  $x \neq \pm 1$ . At  $x = 3$ , both forms give 1. For subtraction, the numerator is  $(x + 1) - 2(x - 1) = 3 - x$ , so the minus sign acts on both terms. Multiplication  $[(x - 1)/(x + 2)] \times [(x + 2)/(x + 1)]$  gives  $(x - 1)/(x + 1)$ , but the original excludes  $x = -2$  and  $x = -1$ . Division  $[(x^2 - 1)/(x^2 + 3x + 2)] \div [(x - 1)/(x + 2)]$  gives 1, yet  $x = -2, -1, 1$  are excluded:  $x = 1$  makes the divisor zero. Finally  $F(x) = 6$  reduces to  $x + 3 = 6$ .

## Exercises 2.1·2.3

---

**2.1** (Foundation) Find  $b$  when  $A=24, h=6$  in  $A=bh/2$ .

---

**2.2** (Foundation) Find  $a$  for  $v=19, u=4, t=5$  in  $v=u+at$ .

---

**2.3** (Foundation) Find  $x$  for  $y=17$  in  $y=3x+2$ .

## Exercises 2.1·2.3

---

**2.1** (Foundation) Find  $b$  when  $A=24, h=6$  in  $A=bh/2$ .

**2.1**  $b=2A/h=48/6$ .

---

**2.2** (Foundation) Find  $a$  for  $v=19, u=4, t=5$  in  $v=u+at$ .

**2.2**  $a=(19-4)/5=3$ .

---

**2.3** (Foundation) Find  $x$  for  $y=17$  in  $y=3x+2$ .

**2.3**  $x=(17-2)/3=5$ .

## Exercise 3.1

---

**3.1** Decide whether this claim is true. Justify with a concrete calculation or counterexample: To rearrange a formula, an operation may be applied to one side only.

## Exercise 3.1

---

**3.1** Decide whether this claim is true. Justify with a concrete calculation or counterexample: To rearrange a formula, an operation may be applied to one side only.

**3.1** False. From  $x+3=7$ , subtracting 3 on both sides gives  $x=4$ . Subtracting it only on the left would give  $x=7$ , which fails the original equation.

Local review guidance: show the chosen method; give the checked result; explain the condition or failed shortcut. This is authored classroom guidance.