

Exercise walk-through

2.1.5

Algebraic Fractions

AQA · GCSE · Mathematics

141. Graphical gradients and area estimates · Higher

Graphical gradients and area estimates · Higher

For a distance curve $d=t^2$, the chord from (1,1) to (3,9) has gradient $(9-1)/(3-1)=4$. A tangent at (2,4) passes through (1,0) and (3,8), giving gradient 4 without calculus. A velocity-time trapezium with endpoint velocities 2 and 6 m/s over 3 s has area $(2+6) \times 3/2 = 12$ m. Smaller strips can improve a curved-area estimate; curvature affects over/underestimation. A cost-versus-quantity tangent through (2,12) and (6,28) gives a local rate $(28-12)/(6-2)=4$ yuan per extra item, linking graph slope to a financial interpretation.

183. Reasoning and check

Reasoning and check

For $F(x) = (x^2 - 9)/(x - 3)$, factor $x^2 - 9 = (x - 3)(x + 3)$. Thus $F(x) = x + 3$ for $x \neq 3$; $F(5) = 8$, but $F(3)$ is undefined, not 6. For addition, $1/(x - 1) + 2/(x + 1) = [(x + 1) + 2(x - 1)]/[(x - 1)(x + 1)] = (3x - 1)/(x^2 - 1)$, with $x \neq \pm 1$. At $x = 3$, both forms give 1. For subtraction, the numerator is $(x + 1) - 2(x - 1) = 3 - x$, so the minus sign acts on both terms. Multiplication $[(x - 1)/(x + 2)] \times [(x + 2)/(x + 1)]$ gives $(x - 1)/(x + 1)$, but the original excludes $x = -2$ and $x = -1$. Division $[(x^2 - 1)/(x^2 + 3x + 2)] \div [(x - 1)/(x + 2)]$ gives 1, yet $x = -2, -1, 1$ are excluded: $x = 1$ makes the divisor zero. Finally $F(x) = 6$ reduces to $x + 3 = 6$.

186. Reasoning and check

Reasoning and check

AQA A4 Higher includes algebraic fractions and valid cancellation. The equation example checks whether a candidate is inside the original domain. Explain each factor cancellation, retain original restrictions and check every equation candidate. These operations extend the existing manipulation example into a complete worked arithmetic sequence.

Exercises 2.1·2.3

2.1 (Higher) For $F(x) = (x^2 - 9)/(x - 3)$, find $F(5)$.

2.2 (Higher) Find $1/(x-1) + 2/(x+1)$ at $x=3$.

2.3 (Higher) How many allowed solutions does $(x^2 - 9)/(x - 3) = 6$ have?

3 Exam-style questions

Exercises 2.1·2.3

2.1 (Higher) For $F(x)=(x^2-9)/(x-3)$, find $F(5)$.

2.1 5 is allowed; after factor cancellation, $F(5)=5+3=8$.

2.2 (Higher) Find $1/(x-1)+2/(x+1)$ at $x=3$.

2.2 $1/2+2/4=1$, agreeing with $(3x-1)/(x^2-1)$.

2.3 (Higher) How many allowed solutions does $(x^2-9)/(x-3)=6$ have?

3 Exam-style questions

2.3 The only reduced-equation candidate is 3; the original denominator is zero there.

Exercise 3.1

3.1 Decide whether this claim is true. Justify with a concrete calculation or counterexample: Cancelling a denominator factor always makes its excluded input valid.

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3.1 Decide whether this claim is true. Justify with a concrete calculation or counterexample: Cancelling a denominator factor always makes its excluded input valid.

3.1 False. $(x^2-1)/(x-1)=x+1$ only for $x \neq 1$. At $x=1$ the original expression is $0/0$ and remains undefined after simplification.

Local review guidance: show the chosen method; give the checked result; explain the condition or failed shortcut. This is authored classroom guidance.