

1 . Number

AQA 8300: Foundation and Higher sections retain their labelled scope

1. Number: the foundations everything else stands on

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Order and calculate with positives, negatives, decimals and fractions; the four operations with formal written methods. Primes, factors, HCF/LCM by prime decomposition; indices, roots, surds (H) and standard form. Fractions $\leftrightarrow$ decimals $\leftrightarrow$ percentages as operators; rounding, estimation and bounds (H).

2. Arithmetic, factors, multiples and primes (N1–N5)

Arithmetic, factors, multiples and primes (N1–N5)

Ordering and symbols: order positive and negative integers, decimals and fractions on a number line; use $=$, $\neq$, $<$, $>$, $\leq$, $\geq$.

3. Arithmetic, factors, multiples and primes (N1–N5)

Arithmetic, factors, multiples and primes (N1–N5)

The four operations (N2): formal written methods for integers, decimals, proper and improper fractions and mixed numbers, all signs — set in context including household finance: profit, loss, cost price, selling price, debit, credit, balance, income tax, VAT, interest rate.

4. Arithmetic, factors, multiples and primes (N1–N5)

Arithmetic, factors, multiples and primes (N1–N5)

Inverse operations and priority (N3): cancellation to simplify; BIDMAS brackets, indices/powers, roots, division/multiplication, addition/subtraction.

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5. Arithmetic, factors, multiples and primes (N1–N5)

Arithmetic, factors, multiples and primes (N1–N5)

Primes and structure (N4): prime numbers, factors, multiples, common factors, common multiples, HCF, LCM; prime factorisation in index form (product notation; unique factorisation). Worked pattern:

6. Arithmetic, factors, multiples and primes (N1–N5)

Arithmetic, factors, multiples and primes (N1–N5)

$200 = 2^3 \times 5^2$ HCF = product of the lowest powers of common primes; LCM = product of the highest powers of all primes.

7. Arithmetic, factors, multiples and primes (N1–N5)

Arithmetic, factors, multiples and primes (N1–N5)

(N5) systematic listing and the product rule for counting (H).

8. Indices, surds, standard form and bounds (N7–N9, N14–N16)

Indices, surds, standard form and bounds (N7–N9, N14–N16)

Indices (N7): laws — $a^m \times a^n = a^{(m+n)}$, $a^m \div a^n = a^{(m-n)}$, $(a^m)^n = a^{(mn)}$, $a^{-n} = 1/a^n$, $a^{(1/2)} = \sqrt{a}$; (H) fractional indices $a^{(m/n)}$ = the n -th root of a^m .

9. Indices, surds, standard form and bounds (N7–N9, N14–N16)

Indices, surds, standard form and bounds (N7–N9, N14–N16)

Surds (N8, H): $\sqrt{12} = \sqrt{(4 \times 3)} = 2\sqrt{3}$ — simplify; rationalise denominators (multiply by the conjugate: $1/(3+\sqrt{2}) \times (3-\sqrt{2})/(3-\sqrt{2}) = (3-\sqrt{2})/7$).

10. Indices, surds, standard form and bounds (N7–N9, N14–N16)

Indices, surds, standard form and bounds (N7–N9, N14–N16)

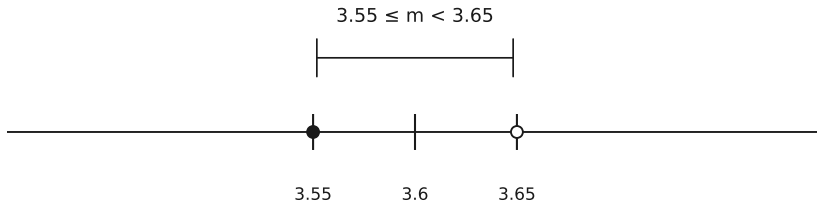
Standard form (N9): $A \times 10^n$, $1 \leq A < 10$, n an integer — add/subtract (match powers), multiply (multiply coefficients and add exponents), or divide (divide coefficients and subtract exponents); interpret calculator displays.

11. Indices, surds, standard form and bounds (N7–N9, N14–N16)

Indices, surds, standard form and bounds (N7–N9, N14–N16)

Estimation (N14): round each value to 1 significant figure, compute, decide over- or under-estimate; check calculations by approximation.

Figure 12: Indices, surds, standard form



$m = 3.6$ to 1 decimal place: closed at the lower bound, open at the upper

Check the labelled values and conditions.

13. Indices, surds, standard form and bounds (N7–N9, N14–N16)

Indices, surds, standard form and bounds (N7–N9, N14–N16)

Rounding and error intervals (N15): to decimal places and significant figures (know not to round mid-calculation); (H) inequality notation for error intervals — 3.6 kg to 1 d.p. means $3.55 \leq m < 3.65$; truncation gives $3.6 \leq m < 3.7$.

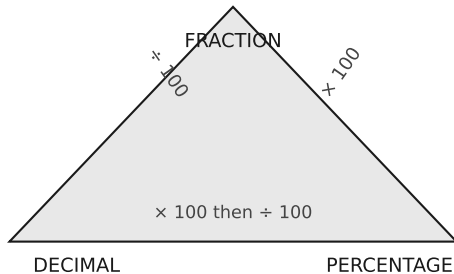
14. Indices, surds, standard form and bounds (N7–N9, N14–N16)

Indices, surds, standard form and bounds (N7–N9, N14–N16)

Bounds (N16, H): upper bound of a length 9 m to the nearest metre is 9.5; in calculations, for positive inputs, an upper product bound uses upper input bounds and an upper quotient bound uses upper numerator divided by lower denominator. Check signs first; these shortcuts need not hold for negative inputs. Track strict endpoints; give the answer to an appropriate degree of accuracy (often the bound width decides).

Figure 15: Fractions, decimals

$1/2 = 0.5 = 50\%$ — every vertex names the same number



Check the labelled values and conditions.

16. Fractions, decimals and percentages (N10–N13, R9 links)

Fractions, decimals and percentages (N10–N13, R9 links)

Conversions (N10): $3.5 = 7/2$, $0.375 = 3/8$; (H) recurring decimal $s \leftrightarrow$
 fractions: $0.\overline{6} = 6/9 = 2/3$; $0.\overline{45}$ (pair) $\rightarrow 45/99 = 5/11$. Method: $x = 0.\overline{6}$, $10x = 6.\overline{6}$, $9x = 6$.

17. Fractions, decimals and percentages (N10–N13, R9 links)

Fractions, decimals and percentages (N10–N13, R9 links)

Fractions as operators (N12): " $\frac{3}{5}$ of 40" = $40 \times \frac{3}{5} = 24$; percentage as multiplier — increase by 15% = $\times 1.15$; decrease by 20% = $\times 0.8$; reverse percentage: find the original before the change by dividing by the multiplier.

18. Fractions, decimals and percentages (N10–N13, R9 links)

Fractions, decimals and percentages (N10–N13, R9 links)

Units (N13): metric conversions for length, area, volume, capacity; compound measures (speed, density, pressure) link to topic 3.

19. Checklist before you call this topic done

Checklist before you call this topic done

Four operations on fractions/mixed numbers and negatives without a calculator; BIDMAS traps. Prime decomposition $\rightarrow$ HCF/LCM; product rule for counting (H). Index laws including negative and fractional; simplify and rationalise surds (H). Standard form arithmetic; rounding and error intervals; upper/lower bounds through a formula (H). FDP conversions including recurring decimals (H); percentage multipliers and reverse percentages.

20. Supported teaching and tier boundary

Supported teaching and tier boundary

8300: Number. Version: Version 1.0, 12 September 2014; first examination 2017.

21. Supported teaching and tier boundary

Supported teaching and tier boundary

Foundation teaching and Higher additions are labelled below. This reference packages the existing native-lesson crosswalk. It does not certify unreviewed specification rows or a whole qualification. Original diagnostics are separate and are not reproduced.

22. Exact arithmetic and estimation · Foundation

Exact arithmetic and estimation · Foundation

Prime factors reveal shared structure. Use the smallest common prime powers for the HCF and the largest for the LCM. Estimate before calculating; use brackets to preserve the order of operations.

23. Exact arithmetic and estimation · Foundation

Exact arithmetic and estimation · Foundation

$$72 = 2^3 \times 3^2, \quad 90 = 2 \times 3^2 \times 5, \quad \text{HCF} = 18$$

24. Exact arithmetic and estimation · Foundation

Exact arithmetic and estimation · Foundation

$72=2^3 \times 3^2$ and $90=2 \times 3^2 \times 5$. Their HCF is $2 \times 9=18$. Make 18 bags with 4 pencils and 5 pens each. Their LCM is $2^3 \times 3^2 \times 5=360$. A prime integer is greater than 1 and has exactly two positive factors. The number 1 is not prime; a composite positive integer greater than 1 has more than two positive factors. Every integer greater than 1 has a unique product of prime factors apart from their order. For example, $60=2^2 \times 3 \times 5$. A factor divides a number; a multiple is the result of multiplying it by an integer. HCF uses common smallest powers, while LCM uses largest powers.

25. Exact arithmetic and estimation · Foundation

Exact arithmetic and estimation · Foundation

The HCF divides both numbers; the LCM is a multiple of both. They answer different questions. A decimal estimate is not an exact fraction.

26. Exact arithmetic and estimation · Foundation

Exact arithmetic and estimation · Foundation

For a non-calculator paper, keep fractions exact and show cancellation. For a calculator paper, enter the full expression and compare with your estimate.

27. Exact arithmetic and estimation · Higher

Exact arithmetic and estimation · Higher

Prime factors reveal shared structure. Use the smallest common prime powers for the HCF and the largest for the LCM. Estimate before calculating; use brackets to preserve the order of operations.

28. Exact arithmetic and estimation · Higher

Exact arithmetic and estimation · Higher

$$a = \prod p_i^{\alpha_i}, \quad b = \prod p_i^{\beta_i}, \quad \text{HCF}(a, b) = \prod p_i^{\min(\alpha_i, \beta_i)}$$

29. Exact arithmetic and estimation · Higher

Exact arithmetic and estimation · Higher

$72=2^3 \times 3^2$ and $90=2 \times 3^2 \times 5$. Their HCF is $2 \times 9=18$. Make 18 bags with 4 pencils and 5 pens each. Their LCM is $2^3 \times 3^2 \times 5=360$. A prime integer is greater than 1 and has exactly two positive factors. The number 1 is not prime; a composite positive integer greater than 1 has more than two positive factors. Every integer greater than 1 has a unique product of prime factors apart from their order. For example, $60=2^2 \times 3 \times 5$. A factor divides a number; a multiple is the result of multiplying it by an integer. HCF uses common smallest powers, while LCM uses largest powers.

30. Exact arithmetic and estimation · Higher

Exact arithmetic and estimation · Higher

The HCF divides both numbers; the LCM is a multiple of both. They answer different questions. A decimal estimate is not an exact fraction.

31. Exact arithmetic and estimation · Higher

Exact arithmetic and estimation · Higher

For a non-calculator paper, keep fractions exact and show cancellation. For a calculator paper, enter the full expression and compare with your estimate.

Figure 32: Exact arithmetic and estimation

$$72 = 2^3 \times 3^2$$

$$90 = 2 \times 3^2 \times 5$$

$$\text{Shared smallest powers: } 2 \times 3^2 = 18$$

Check the labelled values and conditions.

33. Integer indices and standard form · Foundation

Integer indices and standard form · Foundation

Use integer powers, square and cube roots and standard form with $1 \leq a < 10$. In multiplying powers with the same base, add indices; in division, subtract them.

34. Integer indices and standard form · Foundation

Integer indices and standard form · Foundation

$$a^m a^n = a^{m+n}$$

35. Integer indices and standard form · Foundation

Integer indices and standard form · Foundation

$0.000072 = 7.2 \times 10^{-5}$. Also $2^3 \times 2^4 = 2^7 = 128$. The square root of 81 is 9. Check a standard-form answer by writing it out as a decimal. Useful powers include $3^3 = 27$, $4^3 = 64$, $5^3 = 125$, $15^2 = 225$ and $10^6 = 1,000,000$. Integer laws give $2^0 = 1$, $2^{-3} = 1/8$ and $(2^3)^2 = 2^6 = 64$. For standard-form multiplication, $(3 \times 10^5)(4 \times 10^{-3}) = 12 \times 10^2 = 1.2 \times 10^3$. Division gives $(6 \times 10^5)/(2 \times 10^2) = 3 \times 10^3$. For addition, first align exponents: $3 \times 10^4 + 2 \times 10^3 = 3.2 \times 10^4$.

36. Integer indices and standard form · Foundation

Integer indices and standard form · Foundation

Index laws do not turn a sum into a single power: $2^3 + 2^4 = 24$, not 2^7 . Do not round a surd when an exact answer is requested.

37. Integer indices and standard form · Foundation

Integer indices and standard form · Foundation

This Foundation/Core lesson excludes fractional powers and surd rationalisation. Estimate a result before using a calculator and retain the required precision.

38. Indices, surds and standard form · Higher

Indices, surds and standard form · Higher

For the same positive base, multiplication adds indices and division subtracts them. A negative index means reciprocal; a fractional index represents a root. Standard form has $1 \leq a < 10$.

39. Indices, surds and standard form · Higher

Indices, surds and standard form · Higher

$$a^m a^n = a^{m+n}, \quad a^{-n} = \frac{1}{a^n}, \quad a^{m/n} = (\sqrt[n]{a})^m$$

40. Indices, surds and standard form · Higher

Indices, surds and standard form · Higher

$0.000072 = 7.2 \times 10^{-5}$. Also $16^{(3/4)} = (16^{(1/4)})^3 = 2^3 = 8$. Simplify $\sqrt{72} = 6\sqrt{2}$, then rationalise $1/\sqrt{2} = \sqrt{2}/2$. Useful powers include $3^3 = 27$, $4^3 = 64$, $5^3 = 125$, $15^2 = 225$ and $10^6 = 1,000,000$. Integer laws give $2^0 = 1$, $2^{-3} = 1/8$ and $(2^3)^2 = 2^6 = 64$. For standard-form multiplication, $(3 \times 10^5)(4 \times 10^{-3}) = 12 \times 10^2 = 1.2 \times 10^3$. Division gives $(6 \times 10^5)/(2 \times 10^2) = 3 \times 10^3$. For addition, first align exponents: $3 \times 10^4 + 2 \times 10^3 = 3.2 \times 10^4$. For a positive base, $a^{(m/n)} = (\text{the } n\text{th root of } a)^m$. Thus $27^{(2/3)} = 3^2 = 9$ and $16^{(-1/2)} = 1/4$.

41. Indices, surds and standard form · Higher

Indices, surds and standard form · Higher

To estimate $\sqrt{20}$ without a calculator, $4^2 < 20 < 5^2$ gives $4 < \sqrt{20} < 5$; testing $4.5^2 = 20.25$ shows $\sqrt{20}$ is just below 4.5. Do not use a rough estimate as an exact surd answer.

42. Indices, surds and standard form · Higher

Indices, surds and standard form · Higher

Index laws do not turn a sum into a single power: $2^3 + 2^4 = 24$, not 2^7 . Do not round a surd when an exact answer is requested.

43. Indices, surds and standard form · Higher

Indices, surds and standard form · Higher

Check powers of ten against the original quantity. Use surds for exact geometry, and round only the final length when the question asks for a decimal.

Figure 44: Indices, surds and standard form

$$0.000072 = 7.2 \times 10^{-5}$$

7.2 is the coefficient; -5 is the power

The coefficient is between 1 and 10

Check the labelled values and conditions.

45. Accuracy, bounds and compound measures · Higher

Accuracy, bounds and compound measures · Higher

A value rounded to the nearest unit u lies from stated value $-u/2$ up to, but usually not including, stated value $+u/2$. For positive quantities, combine extremes according to the operation.

46. Accuracy, bounds and compound measures · Higher

Accuracy, bounds and compound measures · Higher

$$A = LW, \quad v = \frac{d}{t}$$

47. Accuracy, bounds and compound measures · Higher

Accuracy, bounds and compound measures · Higher

The lengths satisfy $7.95 \leq L < 8.05$ and $4.95 \leq W < 5.05$. Since $A = LW$, $39.3525 \leq A < 40.6525$. For speed d/t , the largest speed uses the largest distance and smallest positive time.

48. Accuracy, bounds and compound measures · Higher

Accuracy, bounds and compound measures · Higher

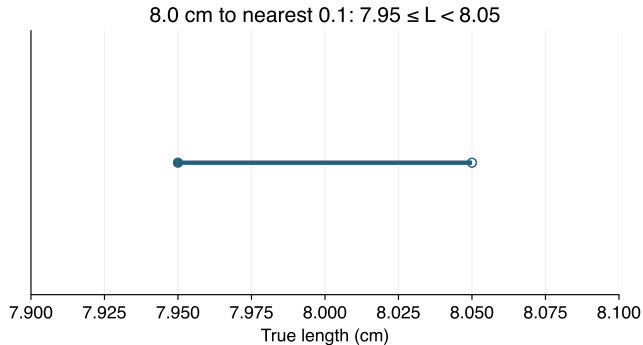
An upper bound is not automatically achieved. Dividing upper distance by upper time does not give the largest speed. Keep enough digits in intermediate calculations.

49. Accuracy, bounds and compound measures · Higher

Accuracy, bounds and compound measures · Higher

Distinguish measurement uncertainty from arithmetic rounding. A sensible reported precision cannot be finer than the measurements justify.

Figure 50: Accuracy, bounds and compound



Check the labelled values and conditions.

51. Signed numbers, place value and operation order · Foundation

Signed numbers, place value and operation order · Foundation

Larger numbers lie farther right on the number line. Adding a negative moves left; subtracting a negative moves right. Multiply or divide signs first, then magnitudes. Evaluate brackets, powers and roots before multiplication/division, then addition/subtraction, working left to right within equal priority.

52. Signed numbers, place value and operation order · Foundation

Signed numbers, place value and operation order · Foundation

$$-12 + 35 - 9 = 14, \quad 18 \div 3 \times 2 = 12$$

53. Signed numbers, place value and operation order · Foundation

Signed numbers, place value and operation order · Foundation

The balance is $-12+35-9=14$ yuan. Also $-4-(-7)=3$ and $(-6)\times(-3)=18$. The reciprocal of -4 is $-1/4$, since their product is 1. For $18\div3\times2$, work left to right to obtain 12. In 3.047, the 4 means four hundredths and the 7 means seven thousandths. For decimal multiplication, first calculate 24×35 : $24\times30+24\times5=720+120=840$. The original 2.4×0.35 has three decimal places in total, giving 0.840. For $5.04\div0.12$ multiply both numbers by 100 to get $504\div12$; $12\times40=480$ leaves 24, so the answer is 42. To order negative fractions, $-3/4=-0.75$ and $-4/5=-0.8$, hence $-4/5<-3/4$.

54. Signed numbers, place value and operation order · Foundation

Signed numbers, place value and operation order · Foundation

The symbols $\leq$ and $\geq$ include equality; $\neq$ means unequal.

55. Signed numbers, place value and operation order · Foundation

Signed numbers, place value and operation order · Foundation

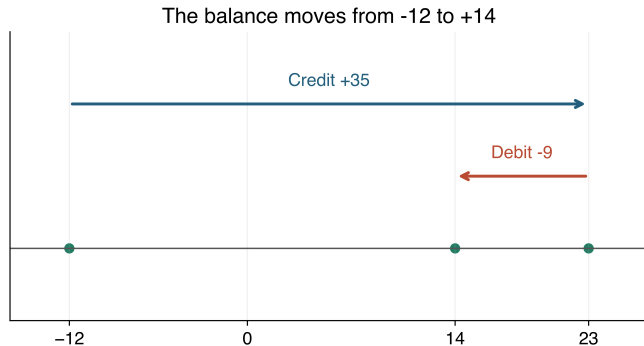
Subtraction is not commutative: $3-8$ and $8-3$ differ. A negative sign outside a square is not inside its base: $-3^2=-9$, but $(-3)^2=9$. Zero has no reciprocal.

56. Signed numbers, place value and operation order · Foundation

Signed numbers, place value and operation order · Foundation

Use a signed starting balance, a positive credit and a negative debit. A shop buying for 48 yuan and selling for 60 makes 12 yuan profit; reversing the prices gives a 12-yuan loss. Estimate the sign before calculating.

Figure 57: Signed numbers, place value



Check the labelled values and conditions.

58. Exact fractions and mixed-number operations · Foundation

Exact fractions and mixed-number operations · Foundation

Convert mixed numbers to improper fractions. Add or subtract using a common denominator; multiply numerators and denominators; divide by a nonzero fraction by multiplying its reciprocal. Cancel common factors, not added terms. These rules also apply to negative fractions.

59. Exact fractions and mixed-number operations · Foundation

Exact fractions and mixed-number operations · Foundation

$$\frac{3}{4} \div \frac{5}{8} = \frac{3}{4} \times \frac{8}{5} = \frac{6}{5}$$

60. Exact fractions and mixed-number operations · Foundation

Exact fractions and mixed-number operations · Foundation

$1\frac{1}{2} - \frac{3}{4} = \frac{6}{4} - \frac{3}{4} = \frac{3}{4}$. Also $(-\frac{2}{3}) \times (\frac{9}{4}) = -\frac{18}{12} = -\frac{3}{2}$ and $(\frac{3}{4}) \div (\frac{5}{8}) = (\frac{3}{4}) \times (\frac{8}{5}) = \frac{6}{5}$. A common denominator gives $\frac{5}{6} + \frac{3}{4} = \frac{10}{12} + \frac{9}{12} = \frac{19}{12}$. Check division by multiplying $\frac{6}{5}$ by $\frac{5}{8}$ to recover $\frac{3}{4}$. Exact multiples of π follow ordinary arithmetic: $3\pi + 2\pi = 5\pi$ and $6\pi/3 = 2\pi$. Leave an answer such as 5π exact when requested; $\pi \approx 3.14$ would introduce approximation. For subtraction, $\frac{5}{6} - \frac{3}{4} = \frac{10}{12} - \frac{9}{12} = \frac{1}{12}$; for a negative mixed number, $-1\frac{1}{2}$ means $-(1 + \frac{1}{2}) = -\frac{3}{2}$.

61. Exact fractions and mixed-number operations · Foundation

Exact fractions and mixed-number operations · Foundation

Adding denominators does not preserve the unit size: $1/2 + 1/3$ is not $2/5$. Cancel only factors of a whole numerator and denominator. A division by zero is undefined.

62. Exact fractions and mixed-number operations · Foundation

Exact fractions and mixed-number operations · Foundation

For a non-calculator question show the common denominator or reciprocal step. Convert $19/12$ to $1\frac{7}{12}$ if a mixed number is requested; round only when the question explicitly needs a decimal.

Figure 63: Exact fractions and mixed-number

Six eighths contain $\frac{6}{5}$ groups of five eighths

$$\frac{3}{4} = \frac{6}{8}$$



Check the labelled values and conditions.

64. Systematic lists and possibility grids · Foundation

Systematic lists and possibility grids · Foundation

Fix one first choice and list every allowed second choice before moving to the next first choice. Use a table to check that no outcome is missing or duplicated. If combinations are forbidden, remove those entries from the list.

65. Systematic lists and possibility grids · Foundation

Systematic lists and possibility grids · Foundation

$\{TA, TB, TM, JA, JB, JM\}$

66. Systematic lists and possibility grids · Foundation

Systematic lists and possibility grids · Foundation

List tea-apple, tea-banana, tea-melon, juice-apple, juice-banana, juice-melon: six choices. Removing tea-melon leaves five. The codes using 1,2,3 once each are 123,132,213,231,312,321: six. These answers follow from complete lists.

67. Systematic lists and possibility grids · Foundation

Systematic lists and possibility grids · Foundation

State whether order matters and whether repetition is allowed. Choosing A then B can be different from B then A for a code, but not for an unordered two-person team. A restriction can make a simple product invalid.

68. Systematic lists and possibility grids · Foundation

Systematic lists and possibility grids · Foundation

Foundation should justify answers with a complete ordered list or grid. Higher may compress a verified structure using the product rule, but must still check restrictions.

69. Systematic lists and the product rule · Higher

Systematic lists and the product rule · Higher

Fix one first choice and list every permitted second choice before moving to the next first choice. A table gives the same structure. When every one of m first choices allows n second choices, the Higher product rule gives mn outcomes. If restrictions change the options, count the allowed rows separately.

70. Systematic lists and the product rule · Higher

Systematic lists and the product rule · Higher

$\{TA, TB, TM, JA, JB, JM\}$

71. Systematic lists and the product rule · Higher

Systematic lists and the product rule · Higher

The list is tea-apple, tea-banana, tea-melon, juice-apple, juice-banana, juice-melon: six outcomes. If tea-melon is unavailable, five remain. For a three-digit code with digits 1,2,3 and no repeats, choose 3 then 2 then 1 possibilities, giving six codes: 123,132,213,231,312,321.

72. Systematic lists and the product rule · Higher

Systematic lists and the product rule · Higher

State whether order matters and whether repetition is allowed. Choosing A then B can be different from B then A for a code, but not for an unordered two-person team. A restriction can make a simple product invalid.

73. Systematic lists and the product rule · Higher

Systematic lists and the product rule · Higher

Foundation should justify answers with a complete ordered list or grid. Higher may compress a verified structure using the product rule, but must still check restrictions.

Figure 74: Systematic lists and the product

A complete grid lists six choices once each

Juice + Apple

Juice + Banana

Juice + Melon

Tea + Apple

Tea + Banana

Tea + Melon

Check the labelled values and conditions.

75. Exact surds and rationalising denominators · Higher

Exact surds and rationalising denominators · Higher

Extract square factors: $\sqrt{a^2b}=a\sqrt{b}$ for $a \geq 0, b \geq 0$. Add only matching root parts. Multiply roots with nonnegative radicands. To rationalise a denominator, multiply numerator and denominator by the same suitable root or conjugate.

76. Exact surds and rationalising denominators · Higher

Exact surds and rationalising denominators · Higher

$$\sqrt{12} + \sqrt{27} = 5\sqrt{3}, \quad \frac{6}{\sqrt{3}} = 2\sqrt{3}$$

77. Exact surds and rationalising denominators · Higher

Exact surds and rationalising denominators · Higher

The side is $\sqrt{12} = \sqrt{4 \times 3} = 2\sqrt{3}$ cm. Thus $\sqrt{12} + \sqrt{27} = 2\sqrt{3} + 3\sqrt{3} = 5\sqrt{3}$. Also $6/\sqrt{3} = 6\sqrt{3}/3 = 2\sqrt{3}$. For $1/(2+\sqrt{3})$, multiply by $(2-\sqrt{3})/(2-\sqrt{3})$: the denominator becomes $4-3=1$, giving $2-\sqrt{3}$. A circle of radius 3 has exact area 9π ; a decimal is an approximation.

78. Exact surds and rationalising denominators · Higher

Exact surds and rationalising denominators · Higher

$\sqrt{(a+b)}$ generally differs from $\sqrt{a}+\sqrt{b}$. Match the radicand before collecting terms. Rationalising changes the form, not the value; multiplying only the denominator changes the value.

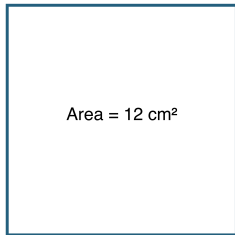
79. Exact surds and rationalising denominators · Higher

Exact surds and rationalising denominators · Higher

AQA N8 Higher requires exact surds and rationalisation. Foundation retains exact fractions and multiples of π ; do not assign this surd lesson to Foundation.

Figure 80: Exact surds and rationalising

An exact root describes the square



$$\text{Side} = \sqrt{12} = 2\sqrt{3} \text{ cm}$$

Check the labelled values and conditions.

81. Terminating decimals and fractions · Foundation

Terminating decimals and fractions · Foundation

Use the place value of the last digit to write a power-of-ten denominator, then simplify. Compare numbers using matching decimal places or a common denominator.

82. Terminating decimals and fractions · Foundation

Terminating decimals and fractions · Foundation

$$0.375 = \frac{375}{1000} = \frac{3}{8}$$

83. Terminating decimals and fractions · Foundation

Terminating decimals and fractions · Foundation

$0.375 = 375/1000 = 3/8$. Also $3.5 = 35/10 = 7/2$ and $0.06 = 6/100 = 3/50$. To order $3/8$ and 0.4 , write 0.375 and 0.400 ; therefore $3/8 < 0.4$.

84. Terminating decimals and fractions · Foundation

Terminating decimals and fractions · Foundation

Keep the place value: 0.06 is six hundredths, not six tenths. A fraction may exceed 1. Simplify the entire numerator and denominator by the same common factor.

85. Terminating decimals and fractions · Foundation

Terminating decimals and fractions · Foundation

AQA N10 Foundation covers terminating decimals and fractions, including ordering. Recurring-decimal conversion is reserved for the Higher lesson.

86. Terminating and recurring decimal conversions · Higher

Terminating and recurring decimal conversions · Higher

A finite decimal uses a power-of-ten denominator before simplifying. For a recurring decimal, multiply by powers of ten so the repeated tails align, then subtract. Use matching decimal places to compare numbers. A displayed rounded decimal need not be the exact fraction.

87. Terminating and recurring decimal conversions · Higher

Terminating and recurring decimal conversions · Higher

$$100x - x = 27, \quad x = \frac{27}{99} = \frac{3}{11}$$

88. Terminating and recurring decimal conversions · Higher

Terminating and recurring decimal conversions · Higher

$0.375 = 375/1000 = 3/8$. For $x = 0.272727\dots$, $100x = 27.272727\dots$, so $99x = 27$ and $x = 27/99 = 3/11$. For $y = 0.16666\dots$, $100y - 10y = 16.666\dots - 1.666\dots = 15$, so $y = 15/90 = 1/6$. In a reduced fraction, a denominator containing only factors 2 and 5 gives a terminating decimal.

89. Terminating and recurring decimal conversions · Higher

Terminating and recurring decimal conversions · Higher

Align the recurring tails before subtracting. 0.333333 is finite and differs from $0.333333\ldots$. A non-recurring prefix needs a second power of ten; blindly dividing every digit block by 99 fails.

90. Terminating and recurring decimal conversions · Higher

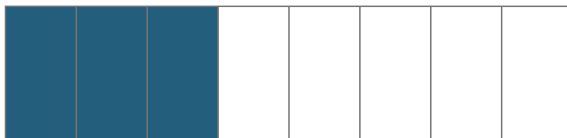
Terminating and recurring decimal conversions · Higher

AQA N10 Higher includes recurring conversion. Both tiers convert and order terminating decimals and fractions. Check a conversion by long division; 3 divided by 8 gives 0.375.

Figure 91: Terminating and recurring decimal

Different exact forms, the same shaded fraction

$$3/8 = 0.375 = 375/1000$$



Check the labelled values and conditions.

92. Metric and compound-unit conversions · Foundation

Metric and compound-unit conversions · Foundation

Convert each dimension. Since $1\text{ m}=100\text{ cm}$, $1\text{ m}^2=10,000\text{ cm}^2$ and $1\text{ m}^3=1,000,000\text{ cm}^3$. Also $1\text{ litre}=1000\text{ cm}^3$, $1\text{ kg}=1000\text{ g}$ and $1\text{ hour}=3600\text{ seconds}$. For a compound unit convert numerator and denominator, keeping the physical quantity unchanged.

93. Metric and compound-unit conversions · Foundation

Metric and compound-unit conversions · Foundation

$$1 \text{ m}^2 = 100^2 \text{ cm}^2, \quad 72 \frac{\text{km}}{\text{h}} = 20 \frac{\text{m}}{\text{s}}$$

94. Metric and compound-unit conversions · Foundation

Metric and compound-unit conversions · Foundation

0.4 m = 40 cm, so the tile area is $40 \times 40 = 1600 \text{ cm}^2$, agreeing with $0.16 \times 10,000$. A 0.002 m^3 container holds $2000 \text{ cm}^3 = 2$ litres. A speed of 72 km/h is $72,000/3600 = 20$ m/s. Write the units at each stage so the conversion can be checked.

95. Metric and compound-unit conversions · Foundation

Metric and compound-unit conversions · Foundation

The area multiplier is the square of the length multiplier; the volume multiplier is its cube. An hour is 60 minutes, not 100. Converting only the numerator of a speed gives an inconsistent unit.

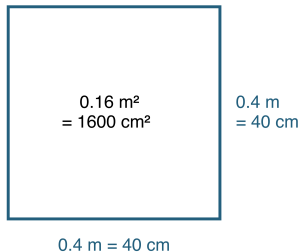
96. Metric and compound-unit conversions · Foundation

Metric and compound-unit conversions · Foundation

AQA N13 and R1 include metric length, area, volume, capacity and compound measures. Use given conversion factors for unfamiliar imperial units; do not guess them.

Figure 97: Metric and compound-unit

Convert both dimensions before multiplying



Check the labelled values and conditions.

98. Estimation, rounding and simple error intervals · Foundation

Estimation, rounding and simple error intervals · Foundation

For a rough check use easy nearby numbers: $50 \times 20 \div 10 = 100$. Decimal places count digits after the point; significant figures start at the first nonzero digit. Round only the final result. Nearest-unit rounding has an interval extending half a unit either way; positive truncation keeps values from the stated value up to the next unit.

99. Estimation, rounding and simple error intervals · Foundation

Estimation, rounding and simple error intervals · Foundation

$$7.95 \leq L < 8.05$$

100. Estimation, rounding and simple error intervals · Foundation

Estimation, rounding and simple error intervals · Foundation

The calculation is about 96.18, consistent with the estimate 100. The number 0.004786 rounds to 0.0048 at 2 significant figures, but to 0.005 at 3 decimal places. If length L rounds to 8.0 cm at 1 decimal place, $7.95 \leq L < 8.05$. If a positive value is truncated to 8.0 at 1 decimal place, $8.0 \leq L < 8.1$ instead. Reported precision must fit the question. Seventeen items packed six per box need three whole boxes, since two boxes hold only twelve items. Rounding $17/6$ to two boxes would fail the physical requirement.

101. Estimation, rounding and simple error intervals · Foundation

Estimation, rounding and simple error intervals · Foundation

For a nearest-0.1 reading of 8.0, each possible value differs from the report by at most 0.05; a claim of 8.08 lies outside the interval. Carry full calculator precision until a final money, length or accuracy requirement is applied.

102. Estimation, rounding and simple error intervals · Foundation

Estimation, rounding and simple error intervals · Foundation

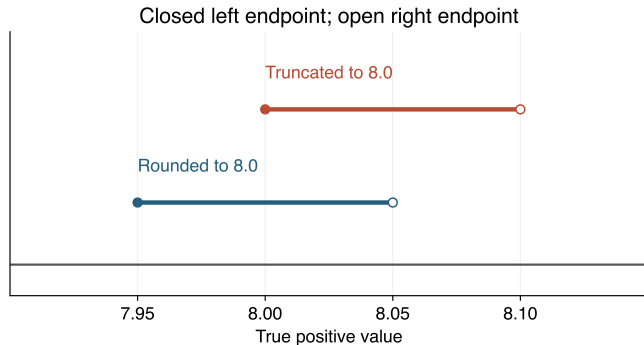
Zeros before the first nonzero digit do not count as significant figures. Rounding and truncation give different intervals. Do not turn an approximate check into an exact answer, or round every intermediate result.

103. Estimation, rounding and simple error intervals · Foundation

Estimation, rounding and simple error intervals · Foundation

AQA N14–N16 require accuracy interpretation at both tiers. Foundation uses simple intervals and limits of accuracy. Higher combines upper and lower bounds for calculated quantities in the separate bounds lesson.

Figure 104: Estimation, rounding and simple



Check the labelled values and conditions.

105. Ratio shares and fraction operators · Foundation

Ratio shares and fraction operators · Foundation

Add the ratio parts to find the whole: 2:5 has 7 equal parts. Divide the total by 7, then multiply by the required part count. A fraction acts as a multiplier; a percentage p acts as $p/100$. Keep part-to-part and part-to-whole comparisons separate.

106. Ratio shares and fraction operators · Foundation

Ratio shares and fraction operators · Foundation

$$84 \times \frac{2}{2+5} = 24$$

107. Ratio shares and fraction operators · Foundation

Ratio shares and fraction operators · Foundation

Each part is $84/7=12$ yuan, giving shares 24 and 60. The first team has $2/7$ of the whole, while its share is $2/5$ of the other share. A $3/4$ portion of 28 is 21. A 120% amount of 35 is $1.2 \times 35 = 42$, so percentages may exceed 100.

108. Ratio shares and fraction operators · Foundation

Ratio shares and fraction operators · Foundation

A ratio 2:5 does not give the first share as $\frac{2}{5}$ of the total. The total has 7 parts. A multiplier over 1 increases a positive quantity; it does not automatically mean a probability.

109. Ratio shares and fraction operators · Foundation

Ratio shares and fraction operators · Foundation

AQA N11/N12 and R3–R8 link ratio parts to fractions and operators. Check the shares add to the stated total and simplify a ratio by dividing every part by the same positive factor.

Figure 110: Ratio shares and fraction

Ratio 2:5 has seven total parts

24 yuan

60 yuan



Check the labelled values and conditions.

111. Try independently

Try independently

A supplier has 84 pencils and 126 pens. Find the greatest number of identical packs using everything. State each pack's contents. A delivery charge of 18.60 CNY is shared equally among six buyers; find each share without rounding intermediate values.

112. Reasoning and check

Reasoning and check

$84 = 2^2 \cdot 3 \cdot 7$ and $126 = 2 \cdot 3^2 \cdot 7$, so the HCF is 42. Each of 42 packs has $84/42 = 2$ pencils and $126/42 = 3$ pens. This is maximal because any pack count must divide both totals. Each delivery share is $s = C/n = (18.60 \text{ CNY})/6 = 3.10 \text{ CNY}$; six shares return the original charge.

113. Try independently

Try independently

A positive rectangle has recorded length 8.0 cm and width 3.0 cm, each to the nearest 0.1 cm. Give tight lower and upper bounds for area and for length divided by width. State endpoint inclusion.

114. Reasoning and check

Reasoning and check

$7.95 \leq l < 8.05$ and $2.95 \leq w < 3.05$, in cm. Positivity makes product and quotient monotonic in the needed directions. Thus $23.4525 \leq A < 24.5525$ in square centimetres. For the dimensionless quotient, $7.95/3.05 < l/w < 8.05/2.95$, or $159/61 < l/w < 161/59$. Neither quotient endpoint is attained, since each requires one excluded upper measurement bound. The area lower bound is attained, since both lower bounds are allowed. Blindly using “upper over lower” without sign and endpoint checks is insufficient.

115. Terms

Terms
prime factor

.

116. Terms

Terms
index .

117. Terms

Terms

lower bound .

118. Terms

Terms
reciprocal .

119. Terms

Terms

improper fraction .

120. Terms

Terms

systematic list .

121. Terms

Terms
surd

.

122. Terms

Terms

terminating decimal

.

123. Terms

Terms

recurring decimal .

124. Terms

Terms	
conversion factor	.

125. Terms

Terms

significant figure

.

126. Terms

Terms
ratio .

Check yourself

Reconstruct the method without looking back

A positive rectangle has recorded length 8.0 cm and width 3.0 cm, each to the nearest 0.1 cm. Give tight lower and upper bounds for area and for length divided by width. State endpoint inclusion.