

Course Companion

AQA · GCSE · Mathematics

Every topic handout in one volume

exercises.lol

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1 · Number

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Number: the foundations everything else stands on

- Order and calculate with positives, negatives, decimals and fractions; the four operations with formal written methods.
- Primes, factors, HCF/LCM by prime decomposition; indices, roots, surds (H) and standard form.
- Fractions $\leftrightarrow$ decimals $\leftrightarrow$ percentages as operators; rounding, estimation and bounds (H).

Arithmetic, factors, multiples and primes (N1–N5)

Ordering and symbols: order positive and negative integers, decimals and fractions on a number line; use $=$, $\neq$, $<$, $>$, $\leq$, $\geq$.

The four operations (N2): formal written methods for integers, decimals, **proper and improper fractions and mixed numbers**, all signs – set in context including household finance: **profit, loss, cost price, selling price, debit, credit, balance, income tax, VAT, interest rate.**

Inverse operations and priority (N3): cancellation to simplify; **BIDMAS** – brackets, indices/powers, roots, division/multiplication, addition/subtraction.

Primes and structure (N4): prime numbers, **factors, multiples, common factors, common multiples, HCF, LCM; prime factorisation** in index form (product notation; unique factorisation). Worked pattern:

- $200 = 2^3 \times 5^2$
- HCF = product of the **lowest** powers of common primes; LCM = product of the **highest** powers of all primes.

Worked example. $84 = 2^2 \times 3 \times 7$ and $120 = 2^3 \times 3 \times 5$, so $\text{HCF} = 2^2 \times 3 = 12$ (lowest powers of the shared primes) and $\text{LCM} = 2^3 \times 3 \times 5 \times 7 = 840$ (highest powers of all primes). Check: $12 \times 840 = 10080 = 84 \times 120$ – the HCF-LCM product identity confirms both.

(N5) systematic listing and the **product rule for counting** (H).

Indices, surds, standard form and bounds (N7–N9, N14–N16)

Indices (N7): laws – $a^m \times a^n = a^{(m+n)}$, $a^m \div a^n = a^{(m-n)}$, $(a^m)^n = a^{(mn)}$, $a^{(-n)} = 1/a^n$, $a^{(1/2)} = \sqrt{a}$; (H) fractional indices $a^{(m/n)}$ = the n-th root of a^m .

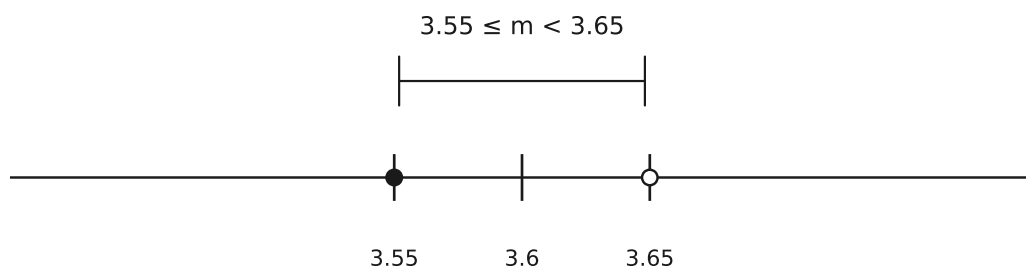
Surds (N8, H): $\sqrt{12} = \sqrt{(4 \times 3)} = 2\sqrt{3}$ – simplify; **rationalise denominators** (multiply

by the conjugate: $1/(3+\sqrt{2}) \times (3-\sqrt{2})/(3-\sqrt{2}) = (3-\sqrt{2})/7$.

Standard form (N9): $A \times 10^n$, $1 \leq A < 10$, n an integer – add/subtract (match powers), multiply (multiply coefficients and add exponents), or divide (divide coefficients and subtract exponents); interpret calculator displays.

Worked example. $(3 \times 10^7) \times (4 \times 10^{-3}) = 12 \times 10^4 = 1.2 \times 10^5$ (coefficient multiplied, exponents added, then renormalised); $(3 \times 10^7) + (4 \times 10^{-3})$ cannot combine until the powers match, so convert first: $30\,000\,000 + 0.004 = 30\,000\,000.004 = 3.0000000004 \times 10^7$ – the tiny term barely moves the large one.

Estimation (N14): round each value to 1 significant figure, compute, decide over- or under-estimate; check calculations by approximation.



$m = 3.6$ to 1 decimal place: closed at the lower bound, open at the upper

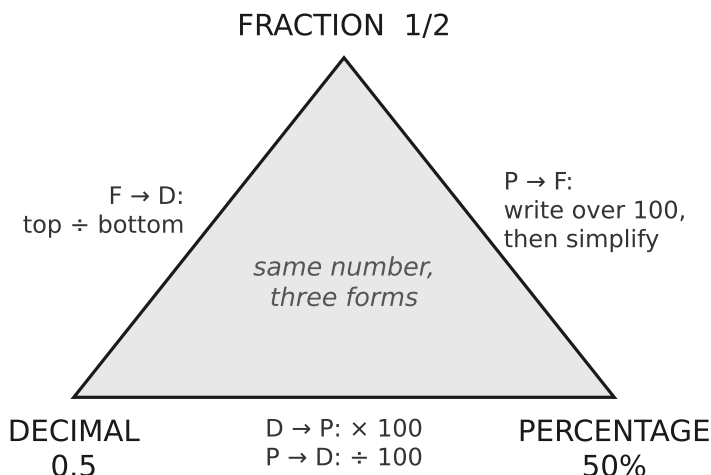
The error interval for 3.6 to one decimal place: closed at 3.55, open at 3.65.

Rounding and error intervals (N15): to decimal places and significant figures (know not to round mid-calculation); (H) inequality notation for error intervals – 3.6 kg to 1 d.p. means $3.55 \leq m < 3.65$; truncation gives $3.6 \leq m < 3.7$.

Bounds (N16, H): upper bound of a length 9 m to the nearest metre is 9.5; in calculations, for positive inputs, an upper product bound uses upper input bounds and an upper quotient bound uses upper numerator divided by lower denominator. Check signs first; these shortcuts need not hold for negative inputs. Track strict endpoints; give the answer to an appropriate degree of accuracy (often the bound width decides).

Worked example: Higher tier. A rectangle has sides 8 m and 5 m, each to the nearest metre: upper area = $8.5 \times 5.5 = 46.75 \text{ m}^2$, lower area = $7.5 \times 4.5 = 33.75 \text{ m}^2$. For the perimeter both OUTSIDE edges grow: upper $P = 2 \times (8.5 + 5.5) = 28 \text{ m}$ – but a QUOTIENT like length $\div$ width needs upper $\div$ lower = $8.5 \div 4.5 = 1.88 \approx 1.9$ (2 s.f.), because dividing by the smaller denominator pushes the answer up.

Fractions, decimals and percentages (N10–N13, R9 links)



The FDP triangle linking fraction, decimal and percentage.

Conversions (N10): $3.5 = 7/2$, $0.375 = 3/8$; (H) **recurring decimals** $\leftrightarrow$ fractions: $0.6 = 6/9 = 2/3$; 0.45 (pair) $\rightarrow 45/99 = 5/11$. Method: $x = 0.6$, $10x = 6.6$, $9x = 6$.

Fractions as operators (N12): “ $3/5$ of 40” $= 40 \times 3/5 = 24$; percentage as multiplier – increase by 15% $= \times 1.15$; decrease by 20% $= \times 0.8$; reverse percentage: find the original before the change by dividing by the multiplier.

Units (N13): metric conversions for length, area, volume, capacity; compound measures (speed, density, pressure) link to topic 3.

Checklist before you call this topic done

- Four operations on fractions/mixed numbers and negatives without a calculator; BIDMAS traps.
- Prime decomposition $\rightarrow$ HCF/LCM; product rule for counting (H).
- Index laws including negative and fractional; simplify and rationalise surds (H).
- Standard form arithmetic; rounding and error intervals; upper/lower bounds through a formula (H).
- FDP conversions including recurring decimals (H); percentage multipliers and reverse percentages.

Algebra

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AQA GCSE Mathematics 8300 topic 3.2 (specification Version 1.0, 12 September 2014, first exams 2017). Foundation rows run A1–A25 in three tiers (basic foundation / additional foundation / Higher only); the Higher-only additions are surds and algebraic fractions (A4), identities and proof (A6), inverse and composite functions with f , fg and f^{-1} notation (A7), perpendicular lines (A9), turning points by completing the square (A11), cubic, reciprocal, exponential and trigonometric graphs (A12), the circle equation with tangents (A16), the quadratic formula (A18), linear/quadratic simultaneity (A19), iteration (A20), set-notation and graphical inequality regions (A22) and quadratic/geometric/surd sequences (A24–A25). The June 2024 Higher set supplies verified anchors below.

Must-know for topic 3.2 (the Formulae Sheet gives only the quadratic formula):

Family	Formulae
quadratic	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$; $a(x + b)^2 + c$ gives the turning point
lines	$m = \frac{y_2 - y_1}{x_2 - x_1}$; perpendicular $m_1 m_2 = -1$; $y - y_1 = m(x - x_1)$
iteration	$x_{n+1} = f(x_n)$ from a rearranged equation, with suffix notation
nth terms	linear $an + b$; quadratic $an^2 + bn + c$ with $a = \frac{1}{2}(\text{second difference})$

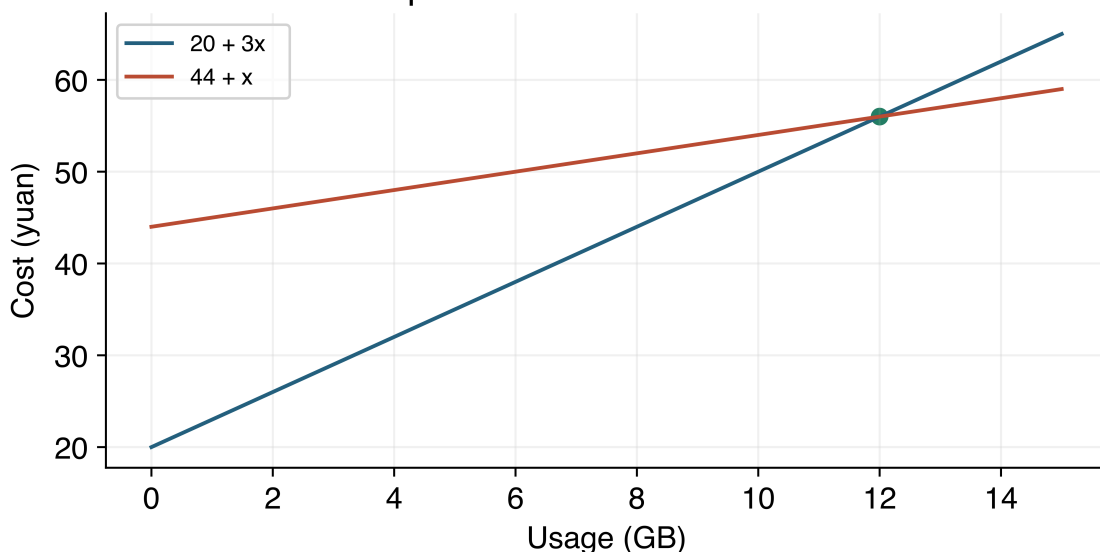
Notation, vocabulary and manipulation

Algebraic notation writes ab for $a \times b$, $3y$ for $y + y + y$, a^2 , a^3 , a^2b , $\frac{a}{b}$ for $a \div b$ – coefficients as fractions, not decimals – and brackets; answers are expected in simplest form without being told. Substitute into formulae including scientific ones (unseen formulae are quoted in the question). The vocabulary – expression, equation, formula, inequality, identity, term, factor – carries meaning: an equation asks which values work; an identity holds for every value.

Simplify and manipulate by collecting like terms, multiplying a single term over a bracket, taking out common factors and using the index laws. Foundation additional adds expanding two binomials and factorising $x^2 + bx + c$ with the difference of two squares. Higher adds surds and **algebraic fractions**: expand and simplify $(3 + \sqrt{2})(1 - 2\sqrt{2}) = 3 - 6\sqrt{2} + \sqrt{2} - 4 = -1 - 5\sqrt{2}$; factorise $6x^2 - 7x - 3 = (3x + 1)(2x - 3)$.

An **identity** argues equivalence for ALL values: $(x + 2)^2 \equiv x^2 + 4x + 4$, and the 3H June 2024 anchor asks exactly this ("it is true for all values of x "). Proofs expand and compare both sides – numeric examples never prove an identity. Rearrange formulae to change the subject ($v = u + at \Rightarrow t = \frac{v-u}{a}$), including from other subjects in words or symbols.

Two plans cost the same at $x = 12$



The plans cost the same where $20 + 3x = 44 + x$: $x = 12$, cost 56 yuan

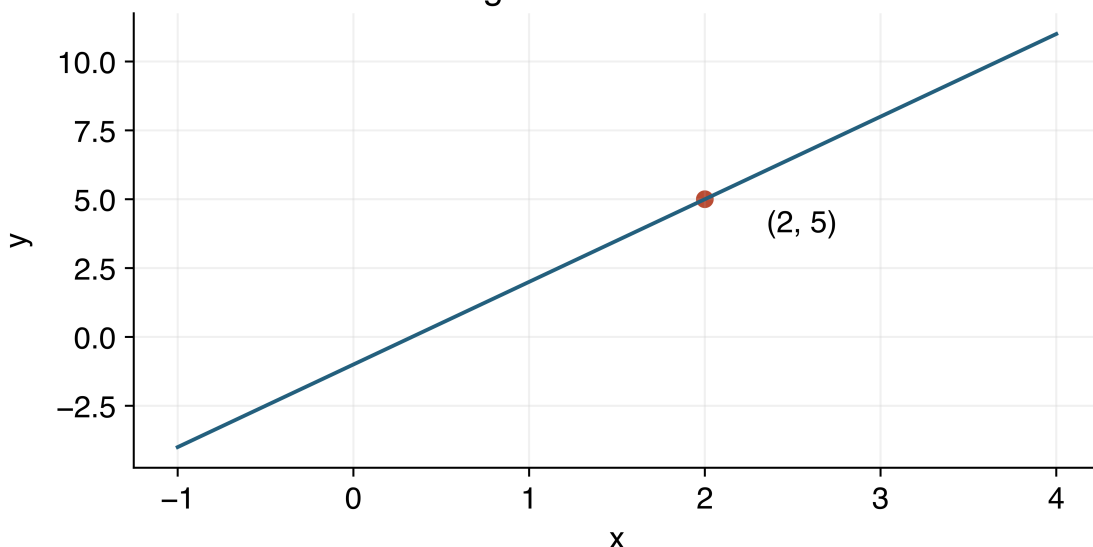
Functions

A function maps inputs to outputs; interpret simple expressions as functions (A7, Foundation). Higher adds the **inverse function** – reverse the operations, or swap and rearrange – and the **composite** $fg(x) = f(g(x))$, apply g first, using $f(x)$, $fg(x)$ and $f^{-1}(x)$ notation. Worked case: $f(x) = 3x - 5$, $g(x) = x^2$: $fg(2) = f(4) = 7$; from $y = 3x - 5$, $x = \frac{y+5}{3}$, so $f^{-1}(x) = \frac{x+5}{3}$. Check $f^{-1}(f(x)) = x$.

Graphs: lines to curves

Coordinates in all four quadrants; plot straight lines from $y = mx + c$. The additional foundation tier identifies parallel lines from equal gradients; Higher finds the equation through two points or one point with a given gradient and identifies perpendicular lines ($m_1m_2 = -1$): through (1,7) and (2,9) the line is $y = 2x + 5$, and the perpendicular through (1,7) is $y - 7 = -\frac{1}{2}(x - 1)$. Identify **gradients** and intercepts graphically AND algebraically (A10).

Point and gradient determine the line



Gradient 3 through (2, 5): $y - 5 = 3(x - 2)$, so $y = 3x - 1$

For quadratics (A11) identify roots, intercepts and turning points graphically; additional foundation deduces roots algebraically; Higher deduces turning points by completing the square : $y = x^2 - 6x + 5 = (x - 3)^2 - 4$ has minimum (3, -4) and the symmetry $x = 3$. Recognise and sketch linear and quadratic graphs (A12); the additional tier adds simple cubics and the **reciprocal** $y = \frac{1}{x}$ ($x \neq 0$); Higher adds exponentials $y = k^x$ ($k > 0$) and $y = \sin x$, $\cos x$, $\tan x$ for angles of any size. Sketch translations and reflections of a given function (A13) – $y = f(x) + a$ up, $y = f(x + a)$ left. Plot and interpret non-standard real-context graphs including kinematic distance/speed/acceleration (A14, reciprocal and exponential at Higher); estimate gradients and areas under graphs and interpret them as rates and distances (A15 – signed velocity-time area gives displacement; adding absolute areas gives total distance).

Higher A16: the circle centred at the origin has $x^2 + y^2 = r^2$, and the tangent at (x_1, y_1) has gradient $-\frac{x_1}{y_1}$ when $y_1 \neq 0$. When both gradients are finite, the radius gradient is $\frac{y_1}{x_1}$ and their product is -1 . At $y_1 = 0$, the tangent is vertical.

Solving equations and inequalities

Solve linear equations in one unknown (A17) – unknown on either side, brackets included: $7x - 22 = 4x + 29$ gives $3x = 51$ and $x = 17$ (the 1H June 2024 anchor Q8). Quadratics (A18): factorise $x^2 + bx + c$ (Foundation additional, including rearrangement), so each **root** comes from a zero factor; Higher adds completing the square and the quadratic formula, with the **discriminant** deciding the root count – $x^2 + 6x + 2 = 0$ gives $x = -3 \pm \sqrt{7}$. Approximate solutions from graphs at every tier.

Worked example: Higher tier. $2x^2 - 5x - 4 = 0$: $b^2 - 4ac = 25 + 32 = 57 > 0$, so two distinct roots; $x = \frac{5 \pm \sqrt{57}}{4}$, and only at the last line $x = 3.14$ or -0.64 (2 d.p.) – keep $\sqrt{57}$ exact until the final answer, and state the accuracy.

Simultaneity (A19): two linear equations algebraically and graphically by **elimination** (Foundation).

Worked example. $3x + 2y = 16$ and $5x - 2y = 8$: the y -terms have opposite signs, so ADD – $8x = 24$, $x = 3$; back-substitute $9 + 2y = 16$, so $y = 3.5$. Same signs: subtract. Nothing matches: scale one equation first.

Higher adds linear/quadratic – $y = 2x + 5$ with $y = x^2$: substituting gives $x^2 - 2x - 5 = 0$, so $x = 1 \pm \sqrt{6}$ – keep both intersections.

Iteration (A20, Higher): solve numerically with suffix notation – from $x^3 - 3x - 1 = 0$, rearrange $x = \sqrt[3]{3x + 1}$ and iterate $x_{n+1} = \sqrt[3]{3x_n + 1}$ from $x_1 = 1$: $x_2 = 1.587$, $x_3 = 1.793$, $x_4 = 1.855$, $x_5 = 1.872$, converging to the root 1.879 (3 d.p.). Show values to a stated accuracy and check the final value back in the original equation.

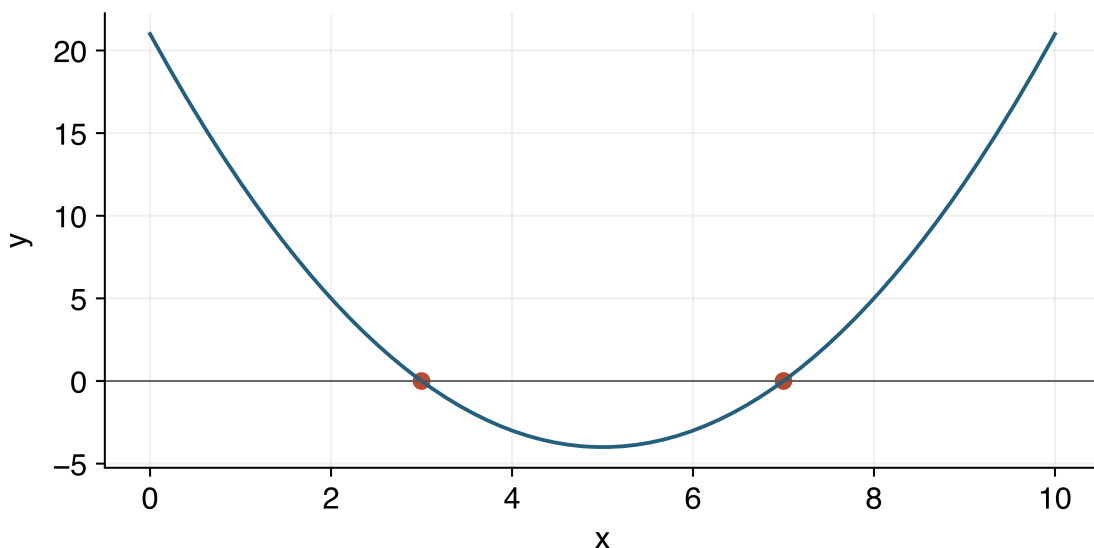
Translate situations into algebra (A21) – derive and solve, including geometry and context problems.

Worked example. Ali is 4 years older than Ben; Cara is twice Ben's age; their ages total 44. Let Ben be x : $x + (x + 4) + 2x = 44$, so $4x = 40$, $x = 10$ – Ben 10, Ali 14, Cara 20; check $10 + 14 + 20 = 44$. Answer the question asked (the ages), not just x .

Inequalities (A22): solve linear inequalities in one variable (Foundation) with number-line representation (open circle strict, closed circle included); Higher solves linear inequalities in one or two variables and quadratic inequalities, with **set notation**

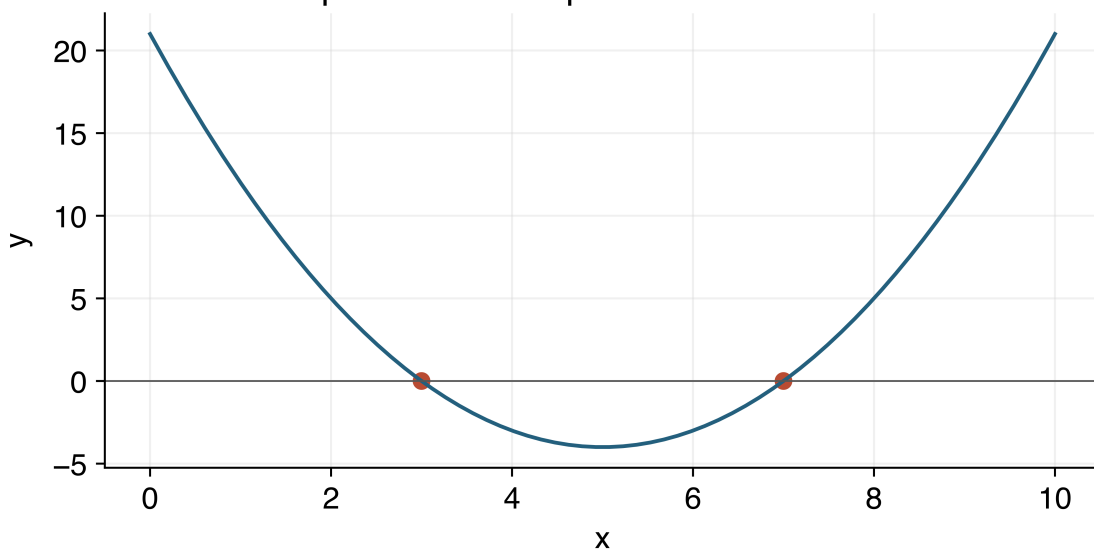
and graphical regions – $x^2 - x - 6 > 0$ factors to $(x - 3)(x + 2) > 0$, so $x < -2$ or $x > 3$, written $\{x: x < -2\} \cup \{x: x > 3\}$; dashed boundary lines for strict inequalities.

Roots occur at $x = 3$ and $x = 7$



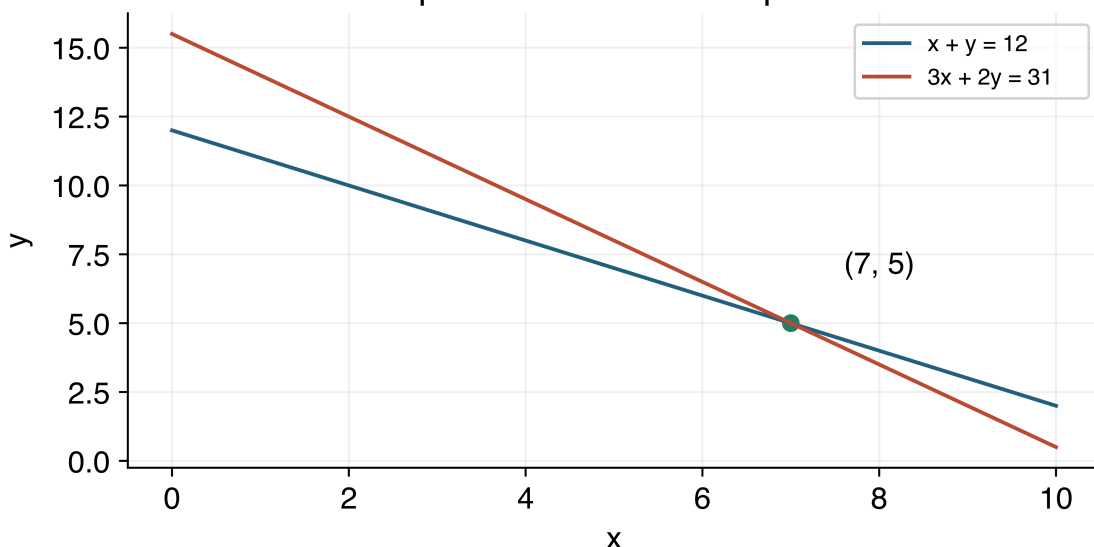
$$x^2 - 10x + 21 = (x - 3)(x - 7): \text{ roots } x = 3 \text{ and } x = 7$$

The quadratic is nonpositive between 3 and 7



$$x^2 - 10x + 21 = (x - 3)(x - 7) \leq 0 \text{ for } 3 \leq x \leq 7$$

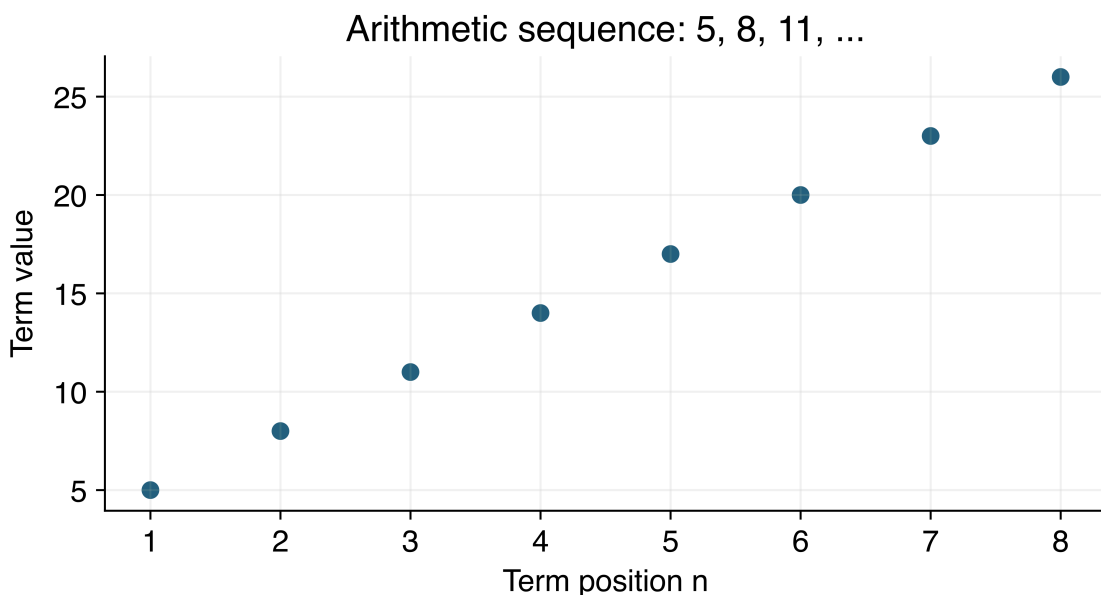
One point satisfies both equations



The solution is the crossing point: $x = 7$, $y = 5$ satisfies both equations

Sequences

Generate terms from term-to-term and position-to-term rules, including patterns and diagrams (A23). Recognise triangular, square and cube numbers and simple arithmetic **progressions** (A24); additional foundation adds Fibonacci-type sequences, quadratic sequences and geometric **progressions** ar^n (r a positive rational); Higher adds other/recursively-defined sequences and surd ratios. Deduce **nth terms** (A25): linear $an + b$ (Foundation), quadratic $an^2 + bn + c$ at Higher – for 5, 9, 17, 29, 45, ... the second difference is 4, so $a = 2$ and the sequence is $2n^2 - 2n + 5$ (check $n = 4$: $32 - 8 + 5 = 29$).



Add 3 each time: n th term $3n + 2$, so the eighth term is 26

Checklist

- Notation ab , $3y$, a^2 , $\frac{a}{b}$; substitution into scientific formulae; vocabulary including identity (A1–A3).
- Collect terms, brackets, common factors, index laws; two binomials and $x^2 + bx + c$ (Foundation); surds and algebraic fractions (Higher) (A4).
- Standard formulae and changing the subject (A5); identities, arguments and proofs (A6).
- Functions with inputs/outputs; inverse and composite with f , fg , f^{-1} notation (A7, Higher).
- Coordinates, plotting, parallel lines (Foundation); equation through two points/point+gradient, perpendicular lines (Higher) (A8–A9).
- Gradients and intercepts both ways (A10); roots, intercepts, turning points – by completing the square at Higher (A11).
- Sketch linear, quadratic, cubic, reciprocal, exponential and trig graphs; translations/reflections; real-context and kinematic graphs; gradient/area interpretation (A12–A15).
- Circle $x^2 + y^2 = r^2$ with tangent gradients (A16, Higher).
- Linear equations both sides (Q8 anchor); quadratics by factorising, formula, completing the square; linear and linear/quadratic simultaneity keeping both roots (A17–A19).

- Iteration with suffix notation to a stated accuracy (A20); modelling and context equations (A21); inequalities with number lines, set notation and graphical regions (A22).
- Sequence types incl. Fibonacci, quadratic, geometric with surd ratios; linear and quadratic nth terms (A23–A25).

Boundary notes: the Formulae Sheet provides the quadratic formula; iteration answers must state the accuracy; graphical inequality conventions (dashed strict, solid included) are required. Worked examples above are adapted from the June 2024 Higher papers (see the meta’s assessment evidence) or original.

Ratio, proportion and rates of change

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AQA GCSE Mathematics 8300 topic 3.3 (specification Version 1.0, 12 September 2014, first exams 2017). Foundation rows R1–R16 run in three tiers; the Higher-only additions are compound units (density, pressure) in numerical AND algebraic contexts (R1), working with percentages above 100% and original-value problems (R9), $k : k^2 : k^3$ similarity ratios (R12), constructing equations for direct and inverse proportion (R13), instantaneous rate of change as the gradient of a tangent (R15), and general iterative processes beyond compound interest (R16). The June 2024 Higher set supplies verified anchors below.

Must-know for topic 3.3:

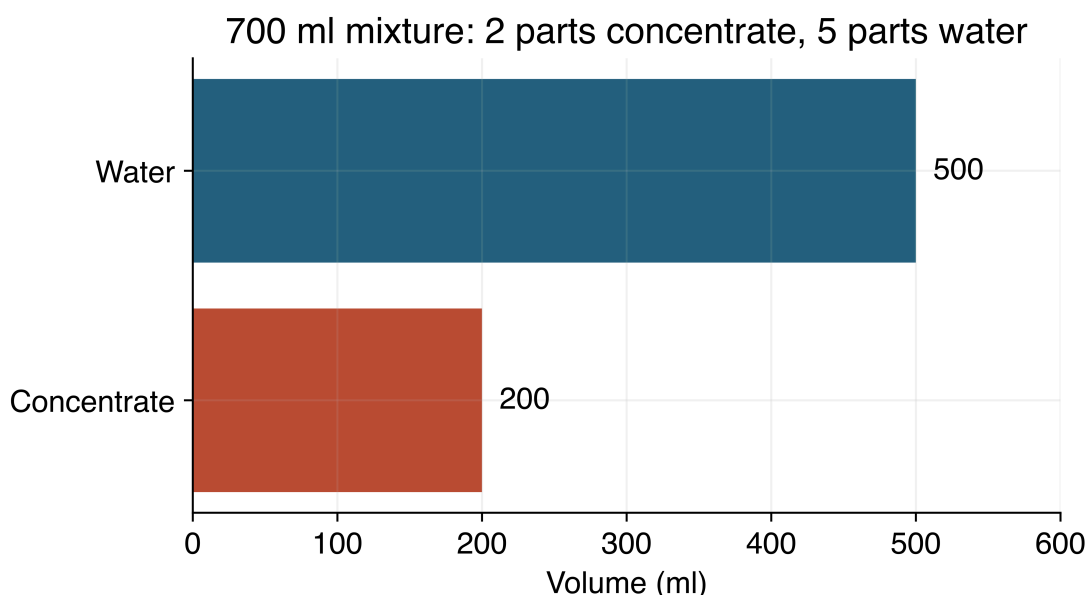
Family	Formulae
multipliers	increase $r\%$: $\times \left(1 + \frac{r}{100}\right)$; decrease: $\times \left(1 - \frac{r}{100}\right)$; original value = final $\div$ multiplier
compound units	speed = $\frac{d}{t}$; density = $\frac{m}{V}$; pressure = $\frac{F}{A}$
proportion	direct $y = kx^n$; inverse $y = \frac{k}{x^n}$ – find k from one pair
growth/decay	$A = P \left(1 \pm \frac{r}{100}\right)^n$; iterative $x_{n+1} = f(x_n)$

Units, ratio notation and sharing

Change freely between related standard units (time, length, area, volume/capacity, mass) and compound units (speed, rates of pay, prices) in numerical contexts; additional foundation adds **density** and **pressure**, and Higher works them in numerical AND algebraic contexts – a cylinder of volume $V = 2x^3$ and mass $m = 5x^2$ has density $\frac{5x^2}{2x^3} = \frac{5}{2x}$.

Ratio notation reduces to simplest form ($45 : 60 = 3 : 4$); express one quantity as a fraction of another (fraction above or below 1); divide a quantity in a part : part or **part : whole ratio** – £350 in 2 : 5 gives £100 and £250. Express a multiplicative relationship as ratio or fraction (R6); proportion as equality of ratios (R7); relate ratios to fractions and LINEAR functions (R8: the unitary method and $y = kx$ are the same ladder). Apply ratio to real contexts – conversion, comparison, scaling, mixing, concentrations, better value/**best-buy** problems (R5): compare unit prices before deciding.

Worked example. Best-buy: a 2.5 kg bag costs £6.40 (unit price $\pounds 6.40 \div 2.5 = \pounds 2.56/\text{kg}$) against a 400 g box at £1.12 ($\pounds 1.12 \div 0.4 = \pounds 2.80/\text{kg}$) – the larger bag is better value, and the comparison only works once BOTH are per kilogram. State the unit prices: the examiner wants the division shown, not just the choice.

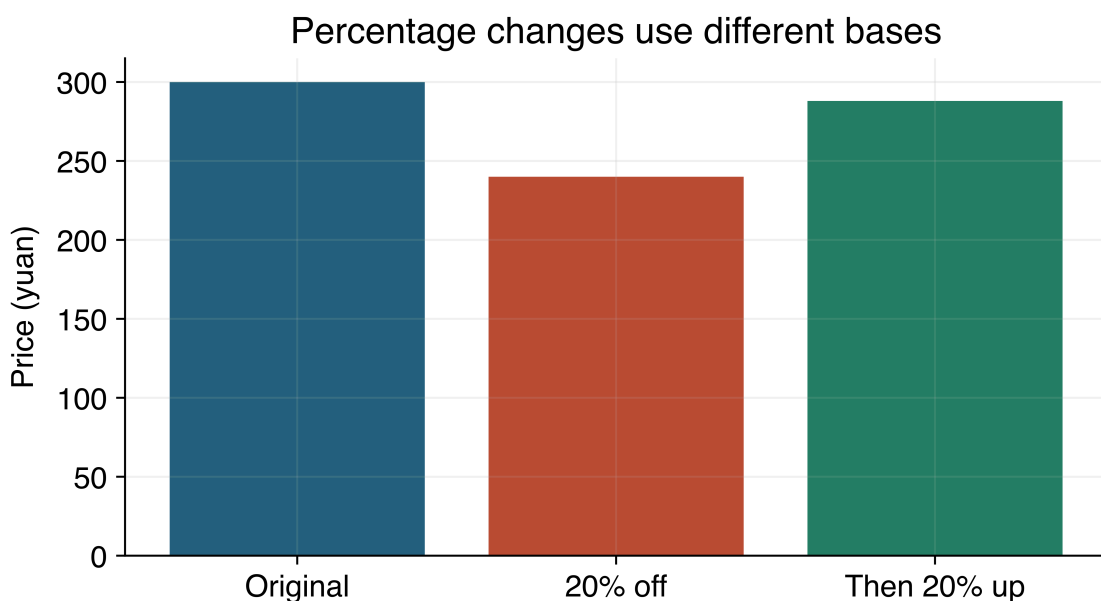


Ratio 2 : 5 has 7 parts; $700 \div 7 = 100$ ml per part, so 200 ml and 500 ml

Percentages and percentage change

Percentage is parts per hundred; interpret percentages and percentage changes as fractions/decimals and MULTIPLICATIVELY (R9). Additional foundation: express one quantity as a percentage of another, compare two quantities, work above 100%, percentage increase/decrease, original-value problems and simple interest. The **multiplier** discipline: +12% is $\times 1.12$; -17% is $\times 0.83$; original value divides the final by the multiplier.

Worked example: 2H June 2024 Q2, adapted. A town's population fell by 12% to 2200. Original = $2200 \div 0.88 = 2500$. The MS's own multiplier check: $2,200,000 \div 0.88 = 2.5 \times 10^6$ in standard form – same division, scaled-up numbers.

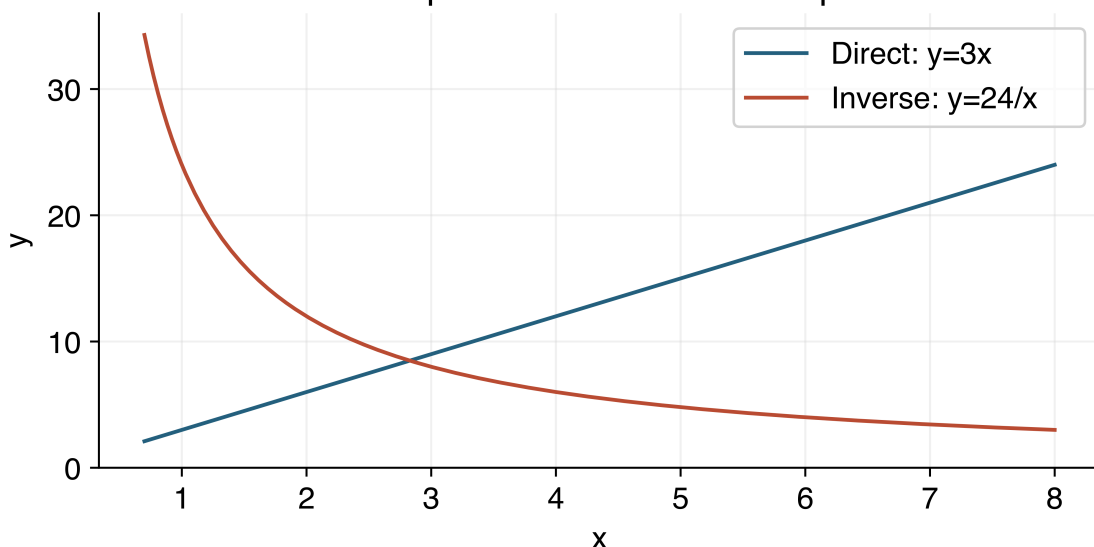


20% off then 20% up gives 288, not 300: the second 20% is of 240

Direct and inverse proportion

Solve direct and inverse proportion problems with graphical and ALGEBRAIC representations (R10). Higher interprets and CONSTRUCTS equations describing proportion (R13): $X \propto Y$ means $X = kY$; X inversely proportional to Y means $X = k/Y$. If $y \propto x^3$ with $y = 24$ at $x = 2$, then the **constant of proportionality** is $k = 3$; if $p \propto \frac{1}{x^2}$ with $p = 8$ at $x = 2$, then $k = 32$ and $p = \frac{32}{x^2}$. Recognise and interpret the graphs: **direct proportion** is a straight line through the origin (higher powers curve upward); **inverse proportion** is a reciprocal-shaped curve (R14 links the gradient of a straight line to a rate of change).

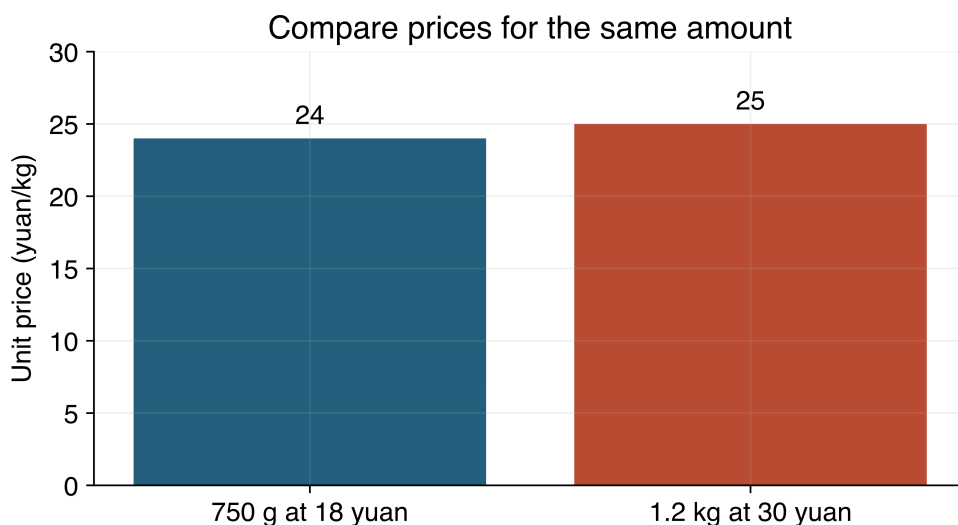
Constant quotient versus constant product



Direct proportion keeps $y \div x$ constant (3); inverse proportion keeps $x \times y$ constant (24)

Rates and compound units

Use compound units – speed, rates of pay, unit pricing (Foundation additional); density and pressure (R11), including comparisons. A 234 km journey in 2 h 36 min averages $234 \div 2.6 = 90$ km/h; £9.20/h for 7.5 h pays £69; best-buy comparisons divide price by quantity and compare like with like.



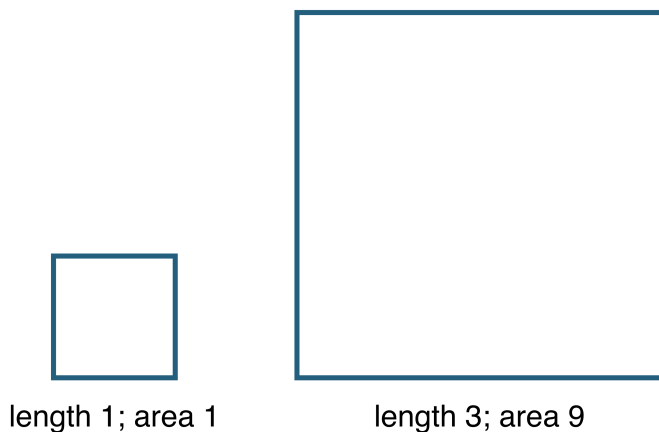
Compare the same amount: $18 \div 0.75 = 24$ yuan/kg beats $30 \div 1.2 = 25$ yuan/kg

Scale factors, similarity and gradients

Use **scale** factors, scale diagrams and maps (R2, incl. geometrical problems). Compare lengths, areas and volumes using ratio notation and scale factors (R12): lengths k : **area scale factors** k^2 : volumes k^3 , with links to similarity including trigonometric ratios; Higher extends to the $k : k^2 : k^3$ ladder in problems – similar statues of heights $3 : 5$ have volumes in ratio $27 : 125$, so a 216 cm^3 smaller statue scales to $216 \times \frac{125}{27} = 1000 \text{ cm}^3$.

Interpret the gradient of a straight-line graph as a **rate of change** (R14) – on a distance-time graph it is speed; on a velocity-time graph the gradient is acceleration and the AREA is distance. Higher (R15) reads the gradient at a point on a CURVE as the instantaneous rate of change: draw the tangent and take its **gradient** as rise/run – and applies average (chord) and instantaneous (tangent) rates in numerical, algebraic and graphical contexts.

A length factor of 3 acts in both area dimensions



A length factor of 3 multiplies area by $3 \times 3 = 9$

Match units before applying the scale

4 cm on map



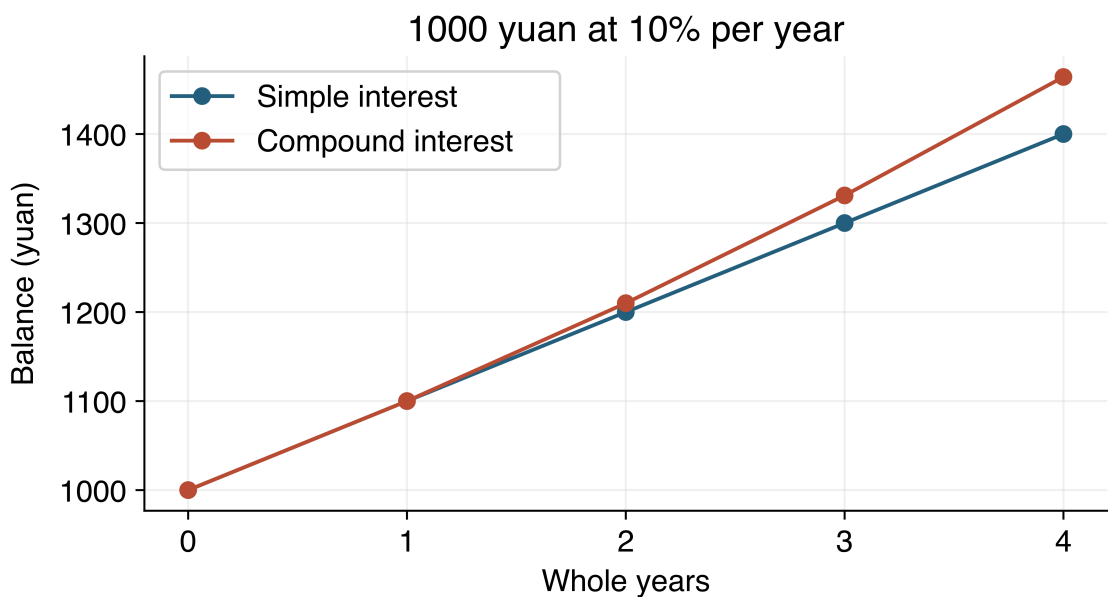
1 km on ground at 1:25,000

$$4 \text{ cm} \times 25\,000 = 100\,000 \text{ cm} = 1 \text{ km}$$

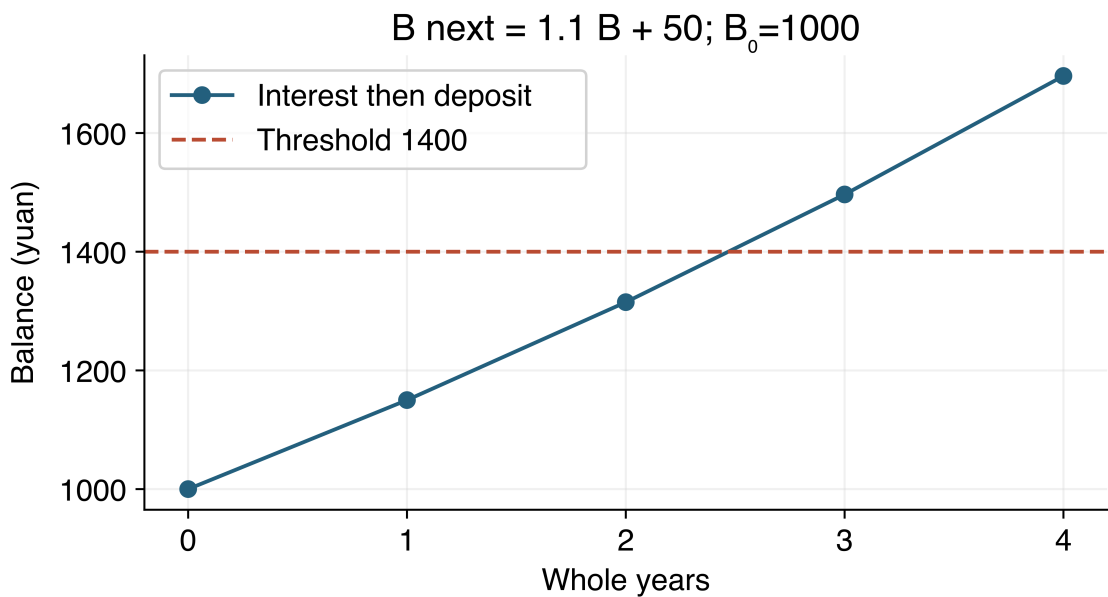
Growth, decay and iterative processes

Set up, solve and INTERPRET growth and decay problems including **compound interest** (R16): $A = P \left(1 \pm \frac{r}{100}\right)^n$. £2000 at 3% compound for 4 years grows to $2000 \times 1.03^4 = £2251.02$; a car worth £12000 losing 15%/year falls to $12000 \times 0.85^3 = £7369.50$.

Worked example: 2H June 2024 Q10, adapted. A house worth £90000 rises 3% then 8%: $90000 \times 1.03 \times 1.08 = £100116$ – above £100000, so yes. The MS's multiplier route: $1.03 \times 1.08 = 1.1124 > 1.111... = 100000/90000$. Higher adds general **iterative processes** : $x_{n+1} = f(x_n)$ cycles for repeated percentages – a quantity gaining 5% each cycle iterates $x_{n+1} = 1.05x_n$, and the limit (if any) solves $x = f(x)$. Interpret answers in context: round sensibly and state the unit.



Simple interest adds 100 each year; compound interest grows by 10% of the new balance



$B_{n+1} = 1.1B_n + 50$: 1000, 1150, 1315, 1496.5, so it first passes 1400 after 3 years

Checklist

- Unit and compound-unit conversions incl. density and pressure, algebraic at Higher (R1, R11).
- Scale factors, diagrams and maps (R2); quantities as fractions; ratio notation, sharing, real-context and best-buy problems (R3–R5).
- Multiplicative relationships as ratio/fraction; proportion as equality of ratios; ratios to fractions and linear functions (R6–R8).
- Percentage multipliers incl. above 100%, change, original value, simple interest (R9; Q2 anchor).
- Direct and inverse proportion graphically and algebraically; interpret and construct proportion equations (R10, R13).
- Gradients as rates of change; chord (average) and tangent (instantaneous) rates (R14–R15).
- Length : area : volume ratios $k : k^2 : k^3$ with similarity links (R12).
- Growth and decay incl. compound interest; general iterative processes with interpretation (R16; Q10 anchor).

Boundary notes: the Formulae Sheet gives nothing for this topic; best-buy comparisons must show unit prices; Higher iterative processes include convergence checks. Worked examples are adapted from the June 2024 Higher papers (see the meta’s assessment evidence) or original.

Geometry and measures

AQA · GCSE · Mathematics

AQA GCSE Mathematics 8300 topic 3.4 (specification Version 1.0, 12 September 2014, first exams 2017). Foundation rows G1–G25 run in three tiers; the Higher-only additions are negative enlargement scale factors (G7), describing change and invariance under combinations of transformations (G8), proving the standard circle theorems (G10), constructing plans and elevations (G13), surface area and volume of spheres, pyramids, cones and composite solids incl. frustums (G17), the $k : k^2 : k^3$ similarity ladder (G19), right and general triangles in 3D (G20), the sine and cosine rules (G22), $\frac{1}{2}ab \sin C$ (G23) and geometric vector proofs (G25). The June 2024 Higher set supplies verified anchors below.

Must-know for topic 3.4:

Family	Formulae
angle sums	triangle 180° ; polygon interior $(n - 2) \times 180^\circ$; exterior turn $\frac{360^\circ}{n}$
circles	$C = 2\pi r = \pi d$; $A = \pi r^2$; arc $= \frac{\theta}{360^\circ} \cdot 2\pi r$; sector $= \frac{\theta}{360^\circ} \cdot \pi r^2$
plane/solid	triangle $\frac{1}{2}bh$; parallelogram bh ; trapezium $\frac{1}{2}(a + b)h$; prism $A_{\text{cross}}L$; cylinder $\pi r^2 h$; cone $\frac{1}{3}\pi r^2 h$; sphere $\frac{4}{3}\pi r^3$, $4\pi r^2$
triangles	$a^2 + b^2 = c^2$; $\sin \theta = \frac{\text{opp}}{\text{hyp}}$, $\cos \theta = \frac{\text{adj}}{\text{hyp}}$, $\tan \theta = \frac{\text{opp}}{\text{adj}}$; sine rule $\frac{a}{\sin A} = \frac{b}{\sin B}$; cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$; area $\frac{1}{2}ab \sin C$
similarity	lengths k : areas k^2 : volumes k^3

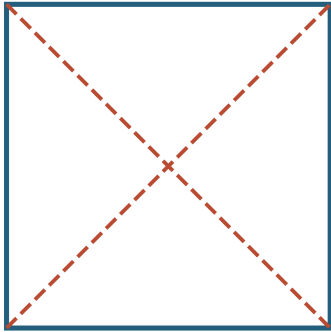
Terms, angle facts and polygons

Use conventional terms and notations – points, lines, vertices, edges, planes, parallel lines, perpendicular lines, right angles, polygons, **regular polygon** (all sides and all angles equal) and polygons with reflection and/or rotation symmetries (G1); label sides and angles of triangles in the standard way (AB a side, angle ABC at vertex B) and draw diagrams from written descriptions. **Corresponding angles** and alternate angles need parallel lines stated, never “Z-angles” (G3): angles at a point sum to 360° , on a line 180° , vertically opposite angles are equal; derive the triangle sum 180° (a line through a vertex parallel to the opposite base transfers the two base angles onto a straight line) and hence the polygon interior sum $(n - 2) \times 180^\circ$ – a pentagon splits into three triangles: 540° ; a regular hexagon has interior angles of 120° and exterior turns of 60° ; an exterior turn of 45° means 8 sides.

Derive and apply the properties and definitions of the special quadrilaterals (G4): a square has 4 equal sides, 4 right angles, 4 **line of symmetry** and rotational order 4; a non-square rectangle has opposite sides equal and equal bisecting diagonals; a rhombus has 4 equal sides and perpendicular bisecting diagonals; a parallelogram’s opposite sides are equal (a diagonal splits it into congruent triangles by ASA); a

kite has two pairs of adjacent equal sides; a trapezium has one pair of parallel sides.
Triangles: isosceles (two equal sides, equal base angles), equilateral, scalene, right-angled, acute, obtuse.

Square diagonals are symmetry axes



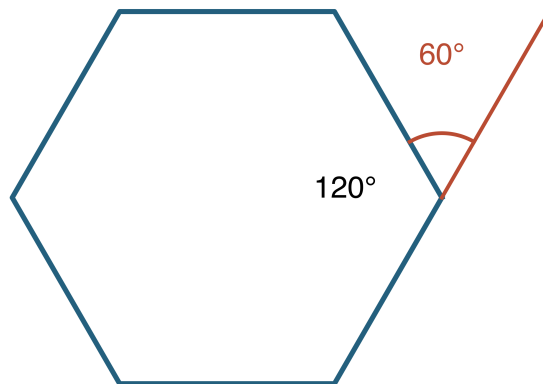
Square



Rectangle

A square's diagonals are lines of symmetry; a rectangle's are not

Regular hexagon: interior 120° , exterior 60°



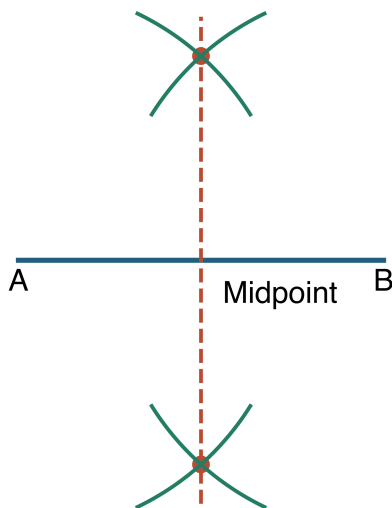
Regular hexagon: exterior angle $360^\circ \div 6 = 60^\circ$, interior angle $180^\circ - 60^\circ = 120^\circ$

Constructions, loci and transformations

Use the standard ruler and compass **constructions** (additional foundation, G2): the **perpendicular bisector** of a segment (equal arcs from both endpoints, radius more than half the segment), the perpendicular to a line from or at a given point, and the bisector of a given angle; construct an angle of 60° from two equal-radius circles (an equilateral triangle). Use these to construct given figures and solve **locus** problems – points equidistant from two points lie on the perpendicular bisector; at fixed distance r from a point, on a circle; equidistant from two lines, on an angle bisector. The perpendicular distance from a point to a line is the shortest (any slanted route is a longer hypotenuse). Keep the construction arcs as evidence.

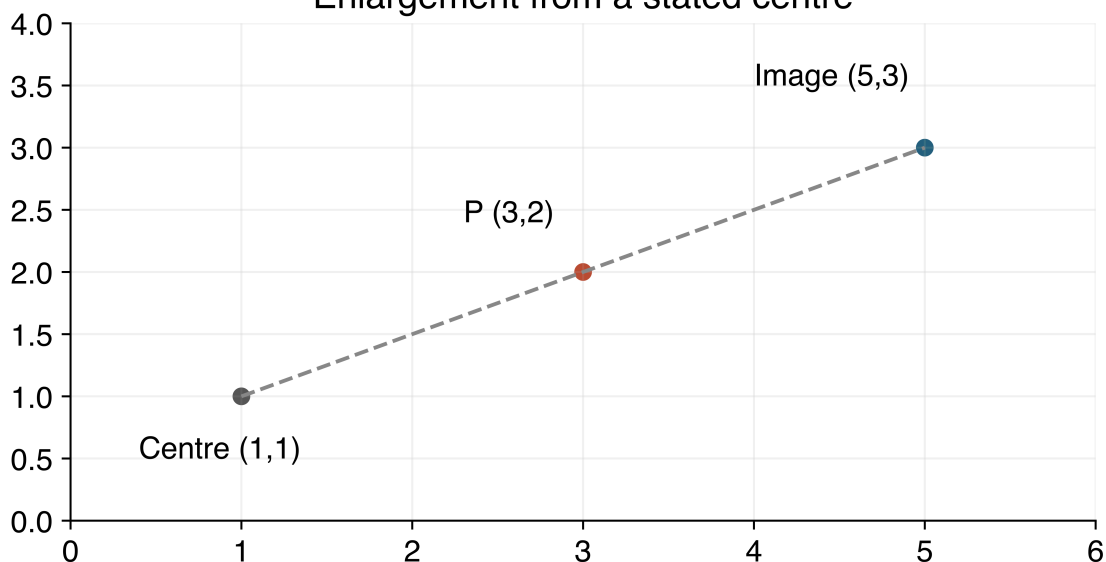
Identify, describe and construct congruent and similar shapes by rotation, reflection, translation and **enlargement** (G7), including on coordinate axes; additional foundation adds fractional scale factors; Higher adds negative ones. For centre $C = (1, 1)$, point $P = (3, 2)$ and factor k , the image is $C + k(P - C)$: $k = 2$ gives $(5, 3)$, $k = \frac{1}{2}$ gives $(2, 1.5)$, and the Higher $k = -2$ gives $(-3, -1)$, on the opposite side of the centre. Higher (G8) describes the changes and **invariant** properties achieved by combinations of rotations, reflections and translations: lengths and angles are preserved (isometries); two reflections in parallel lines $x = 0$ then $x = 2$ send $(3, 2)$ to $(-3, 2)$ then $(7, 2)$ – a translation by $(4, 0)$; in intersecting lines they give a rotation through twice the directed angle between the mirrors. Translations are written with column-vector notation.

Equal compass radii: join the two arc intersections



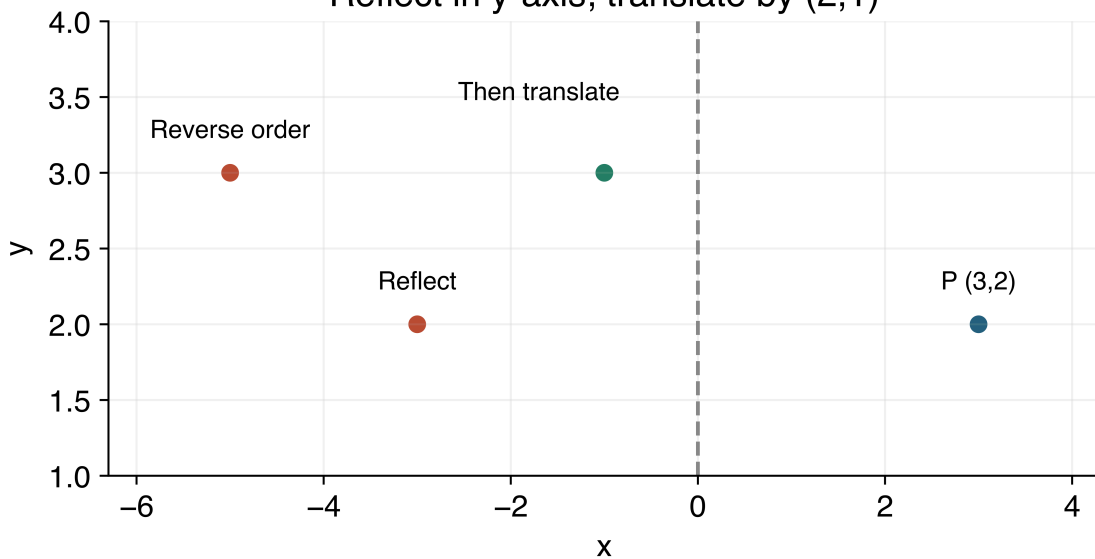
Equal compass arcs from A and B cross twice; the line through the crossings is the perpendicular bisector

Enlargement from a stated centre



Scale factor 2 from (1, 1): the image is twice as far from the centre as P

Reflect in y-axis; translate by (2,1)



Order matters: reflect then translate gives $(-1, 3)$; translate then reflect gives $(-5, 3)$

Congruence, similarity and circle theorems

Apply the basic **congruence** criteria for triangles – SSS, SAS, ASA, RHS (G5) – matching vertices in order; AAA gives similarity, not congruence, and SSA is not a criterion.

Worked example: 2H June 2024 Q12, adapted. Two triangles share three equal corresponding sides. The one-word congruence reason: **SSS**.

Apply angle facts, triangle congruence, similarity and quadrilateral properties to conjecture and derive results, including Pythagoras' theorem and the equality of the base angles of an isosceles triangle, and use known results to obtain simple proofs (G6). For isosceles ABC with $AB = AC$ and D the **midpoint** of BC : triangles ABD and ACD agree in $AB = AC$, $BD = DC$ and shared AD , so SSS gives congruence and matching base angles $ABC = BCA$; the two angles at D are equal and sum to 180° , so each is 90° . Pythagoras follows from four congruent right triangles around a tilted square inside a square of side $a + b$: $(a + b)^2 = 4 \cdot \frac{ab}{2} + c^2$, hence $a^2 + b^2 = c^2$.

Identify and apply circle definitions and properties (G9): centre, radius, chord, diameter ($d = 2r$), **circumference**, with additional foundation adding tangent, arc, **sector** and **segment** – a sector is bounded by two radii and an arc; a segment lies between a chord and an arc. The perpendicular from the centre to a chord bisects it: a chord 3 cm from the centre of a radius-5 circle has length $2\sqrt{5^2 - 3^2} = 8$ cm.

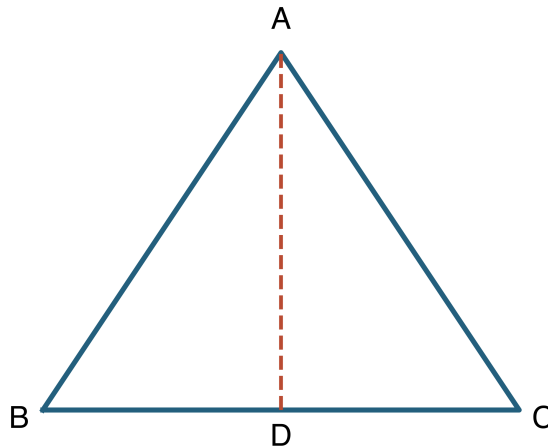
Worked example: 2H June 2024 Q1, adapted. Name each shaded region from: arc, chord, sector, segment, tangent – the sliver between a chord and the arc is the segment; the pie-slice between two radii and their arc is the sector.

Higher (G10) applies and PROVES the standard circle theorems: the **angle at the centre** is twice the angle at the circumference on the same arc (prove with two isosceles triangles on equal radii: $\angle AOB = 360^\circ - (180^\circ - 2\alpha) - (180^\circ - 2\beta) = 2(\alpha + \beta)$); the angle in a semicircle is 90° ; angles in the same segment are equal; opposite angles of a **cyclic quadrilateral** sum to 180° ; the tangent at any point is perpendicular to the radius there; tangents from an external point are equal (RHS on the two contact triangles); the perpendicular from the centre to a chord bisects it; and the **alternate segment theorem** – the tangent-chord angle equals the angle in the opposite segment (a tangent-chord angle of 35° gives 35° in that segment).

Worked example: 1H June 2024 Q22, adapted. Two tangents from P touch a circle at A and B ; angle $APB = 24^\circ$. Draw AO and BO : the quadrilateral $OAPB$ has two 90° contact angles, so the angle at the centre is $\angle AOB = 360^\circ - 90^\circ - 90^\circ - 24^\circ = 156^\circ$ (obtuse, not reflex). The angle at the circumference on the same arc is half of it: $x = \frac{156^\circ}{2} = 78^\circ$ (the mark scheme's $2x = 156^\circ$).

Congruence proves equal base angles

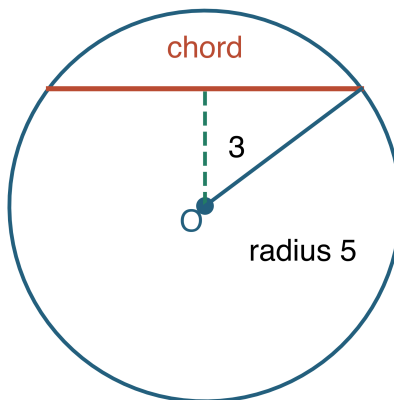
$$AB = AC$$



$$BD = DC; AD \text{ shared} \rightarrow \text{SSS}$$

$AB = AC$, $BD = DC$ and AD shared: SSS congruence, so the base angles are equal

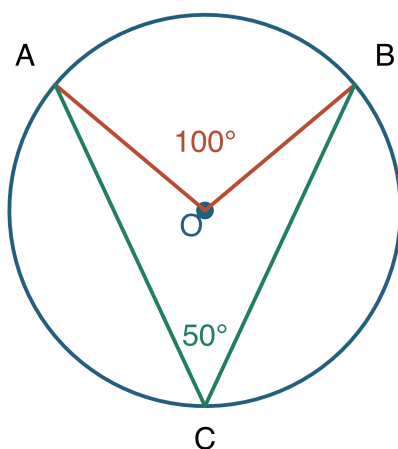
Circle of radius 5: distances use one consistent scale



Perpendicular bisects chord: half-length 4

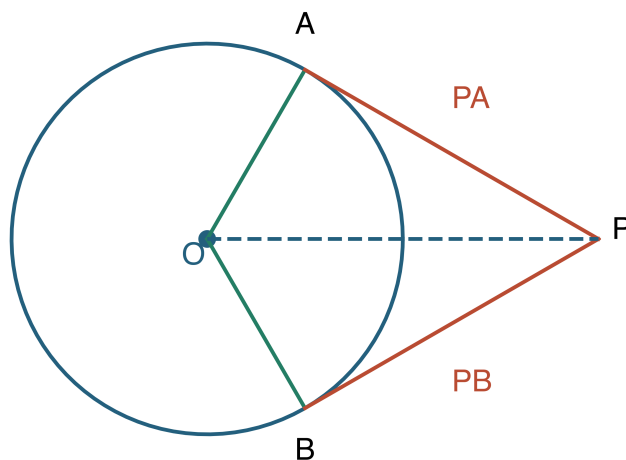
Radius 5, distance 3: half the chord is $\sqrt{5^2 - 3^2} = 4$, so the chord is 8

Angles on the same upper arc: centre = twice circumference



The angle at the centre, 100° , is twice the angle at the circumference, 50°

Shared OP and equal radii $\rightarrow$ RHS $\rightarrow PA = PB$



Equal radii, shared OP and right angles give congruent triangles (RHS), so $PA = PB$

Mensuration: perimeter, area and volume

Use standard units of measure and related concepts (G14). Identify the faces, surfaces, edges and vertices of cubes, cuboids, prisms, cylinders, pyramids, cones and spheres (G12); interpret – and at Higher construct – plans and elevations of 3D shapes (G13): for a $4 \times 3 \times 2$ cuboid the plan is 4×3 , the front **elevation** 4×2 , the side elevation 3×2 . Measure line segments and angles in geometric figures, including interpreting maps and scale drawings and use of **bearing** (G15): three-figure bearings measured clockwise from north – east 090° , and the reverse of 070° is $070^\circ + 180^\circ = 250^\circ$; on a 1 : 10000 map, 3 cm represents 300 m.

Know and apply the area formulae for triangles ($\frac{1}{2}b \times$ **perpendicular height**), parallelograms and trapezia, and volume of cuboids and other right prisms including cylinders (G16): a trapezium with parallel sides 6 and 10 and height 4 has area $\frac{1}{2}(6 + 10) \times 4 = 32 \text{ cm}^2$; a triangular prism with end area 12 cm^2 and length 10 cm has volume 120 cm^3 . Know the circle formulae (G17): $C = 2\pi r = \pi d$, $A = \pi r^2$; a radius-4 circle has circumference 8π and area 16π ; the semicircle on it has area 8π and perimeter $4\pi + 8$; an annulus with radii 5 and 3 has area $\pi(25 - 9) = 16\pi$. Additional foundation: surface area and volume of spheres, pyramids, cones and composite solids, incl. **frustum** – a cone with $r = 3$, $h = 4$ has slant 5, volume 12π , curved area 15π and total 24π ; a sphere with $r = 3$ has volume 36π and surface 36π ; a frustum from a 6, 8-cone (slant 10) minus a similar 3, 4-cone (slant 5) has volume $96\pi - 12\pi = 84\pi$. Solutions in terms of π may be asked for.

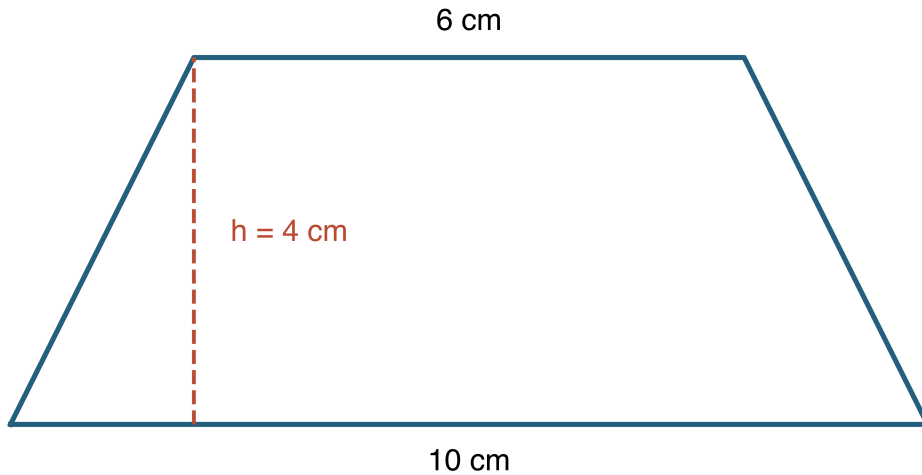
Calculate arc lengths, angles and areas of sectors (G18, additional foundation): at $r = 6$ and $\theta = 120^\circ$ the fraction is $\frac{1}{3}$, so the arc is 4π and the sector area is 12π ; a sector of radius 6 and area 9π has fraction $\frac{1}{4}$, so $\theta = 90^\circ$.

Worked example: 1H June 2024 Q12, adapted. A sector of radius 12 cm has angle 60° : arc $= \frac{60}{360} \times 2 \times 12 \times \pi = 4\pi \text{ cm}$ – show the full-circle quantity and the fraction before multiplying.

Apply congruence and similarity, including the relationships between lengths, areas and volumes in similar figures (G19; Higher): the $k : k^2 : k^3$ ladder with **scale factor** k .

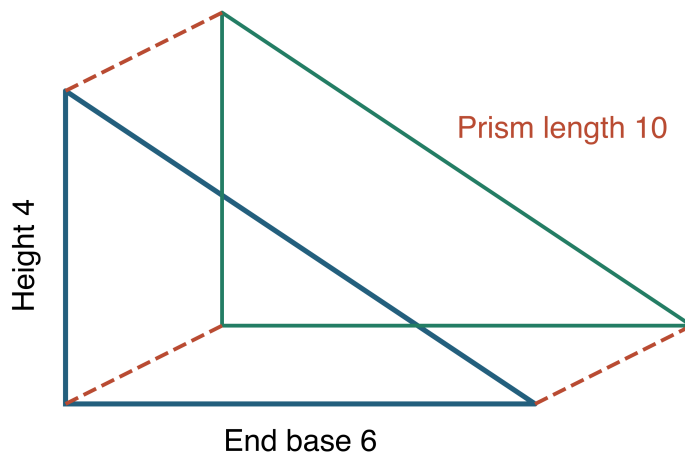
Worked example: 2H June 2024 Q25, adapted. A right-angled triangular prism has edge ratio $a : b : c : d = 3 : 4 : 5 : 12$ and volume 1125 cm^3 . Write the edges as $3k, 4k, 5k, 12k$: volume $= \frac{1}{2}(3k)(4k)(12k) = 72k^3 = 1125$, so $k^3 = 15.625$ and $k = 2.5$. Total edge length $= 2(3 + 4 + 5)k + 3 \times 12k = 60k = 150 \text{ cm}$ (two of each cross-section edge, three length edges).

Trapezium area = average parallel sides $\times$ height



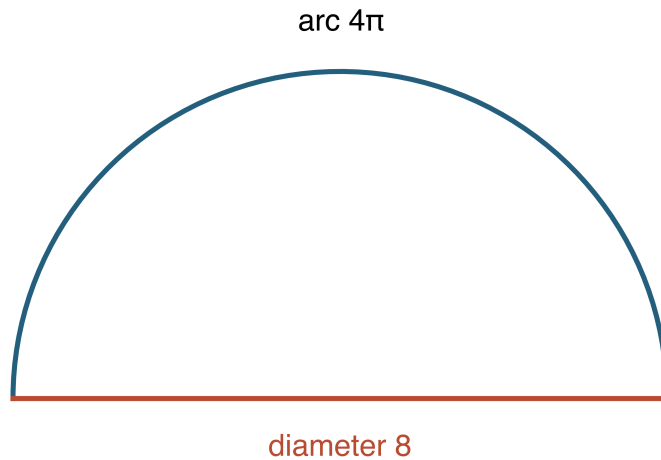
$$\text{Area} = \frac{1}{2}(6 + 10) \times 4 = 32 \text{ cm}^2$$

Constant triangular end area 12 $\rightarrow$ volume 120



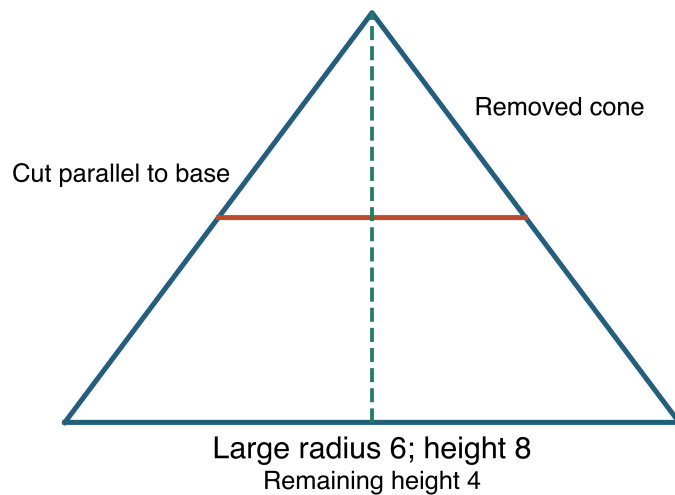
$$\text{End area } \frac{1}{2} \times 6 \times 4 = 12, \text{ volume } 12 \times 10 = 120$$

Semicircle perimeter: curved arc + straight diameter



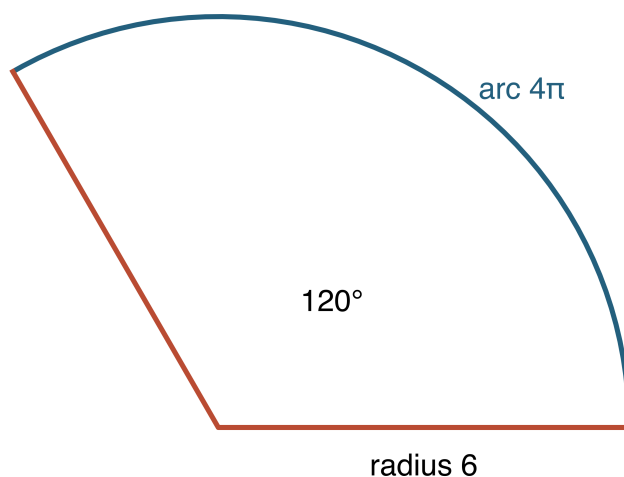
$$\text{Perimeter} = \text{curved arc} + \text{diameter} = 4\pi + 8$$

Frustum volume = whole cone – removed similar cone



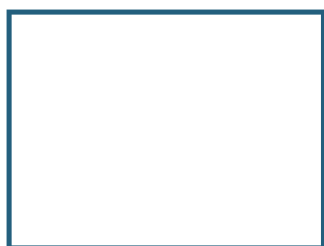
Frustum = whole cone – small cone; the small cone has half the height, so $\frac{1}{8}$ of the volume

One third of a turn: sector area 12π

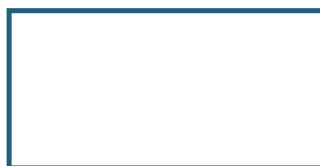


120° is one third of a turn: $\text{arc} = \frac{1}{3} \times 12\pi = 4\pi$, $\text{area} = \frac{1}{3} \times 36\pi = 12\pi$

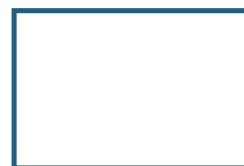
Cuboid width 4, depth 3, height 2



Plan: 4×3



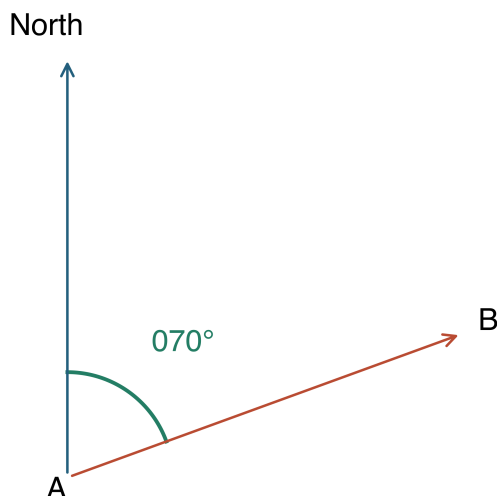
Front: 4×2



Side: 3×2

A $4 \times 3 \times 2$ cuboid: each view shows two of its three dimensions

Clockwise from north at A; reverse bearing at B is 250°



Bearings run clockwise from north: 070° from A, so the back bearing from B is $070^\circ + 180^\circ = 250^\circ$

Pythagoras and trigonometry

Know Pythagoras' theorem $a^2 + b^2 = c^2$ and the trigonometric ratios (G20) – $\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$, $\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$, $\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$ – and apply them to find angles and lengths in right-angled triangles in 2D figures; Higher applies them (and, where possible, general-triangle rules) in 2D AND 3D. A 3-4-5 ladder footed 3 m out reaches $c = \sqrt{3^2 + 4^2} = 5$ m. In 3D, isolate the right triangle: a $3 \times 4 \times 12$ cuboid has base diagonal $\sqrt{9 + 16} = 5$, space diagonal $\sqrt{25 + 144} = 13$, and the space diagonal meets the base at $\tan \alpha = \frac{12}{5}$, so $\alpha \approx 67.4^\circ$.

Worked example: 2H June 2024 Q18, adapted. In a right triangle the hypotenuse is 23.7 and x is the side **adjacent** to the marked angle 52° : $\cos 52^\circ = \frac{x}{23.7}$ (a sine-rule rearrangement $\frac{x}{\sin 38^\circ} = \frac{23.7}{\sin 90^\circ}$ also works), so $x = 23.7 \cos 52^\circ = 14.6$ (1 dp).

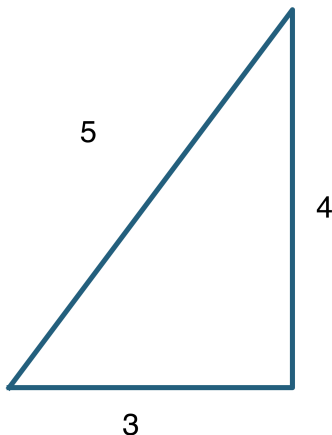
Know the **exact value** of $\sin \theta$ and $\cos \theta$ for $\theta = 0^\circ, 30^\circ, 45^\circ, 60^\circ, 90^\circ$ and of $\tan \theta$ for $0^\circ, 30^\circ, 45^\circ, 60^\circ$ (G21, additional foundation) – derive them from the bisected equilateral triangle $(1, \sqrt{3}, 2)$ and the right isosceles triangle $(1, 1, \sqrt{2})$: $\sin 30^\circ = \frac{1}{2}$, $\cos 30^\circ = \frac{\sqrt{3}}{2}$, $\tan 30^\circ = \frac{1}{\sqrt{3}}$, $\sin 45^\circ = \cos 45^\circ = \frac{\sqrt{2}}{2}$, $\tan 60^\circ = \sqrt{3}$; $\tan 90^\circ$ is undefined.

Worked example: 1H June 2024 Q25, adapted. Show that $6 \sin 30^\circ + 2 \cos 30^\circ \times 4 \tan 30^\circ$ is an integer: $6 \times \frac{1}{2} + \sqrt{3} \times \frac{4}{\sqrt{3}} = 3 + 4 = 7$ – the surds cancel before any rounding.

Higher knows and applies the sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ and the cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$ (G22), and the area formula $A = \frac{1}{2}ab \sin C$ with the **included angle** between a and b (G23). Sides 6 and 8 enclosing 60° give $c^2 = 36 + 64 - 96 \times \frac{1}{2} = 52$ so $c = 2\sqrt{13}$, and area $\frac{1}{2} \times 6 \times 8 \times \frac{\sqrt{3}}{2} = 12\sqrt{3}$. Sine rule: with $a = 4$ opposite $A = 30^\circ$ and $B = 45^\circ$, $b = \frac{4 \sin 45^\circ}{\sin 30^\circ} = 4\sqrt{2}$. An inverse sine may give an acute angle and an obtuse supplement – check the angle sum before choosing.

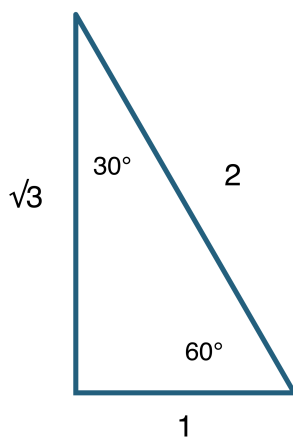
Worked example: Higher tier. $a = 7$, $b = 10$, $A = 40^\circ$: $\sin B = \frac{10 \sin 40^\circ}{7} = 0.918$, so $B = 66.7^\circ$ OR 113.3° – the calculator shows only the acute one. Both keep the angle sum legal ($C = 73.3^\circ$ or 26.7°), so TWO triangles fit the data and the ambiguous answer must be given. Ambiguity can only arise when the given opposite side is the SHORTER one ($a < b$) and A is acute – a bigger side faces a bigger angle, so $a \geq b$ forces $B \leq A$ and kills the supplement; a ten-second sketch settles it before any arithmetic.

Label the known sides before choosing a rule



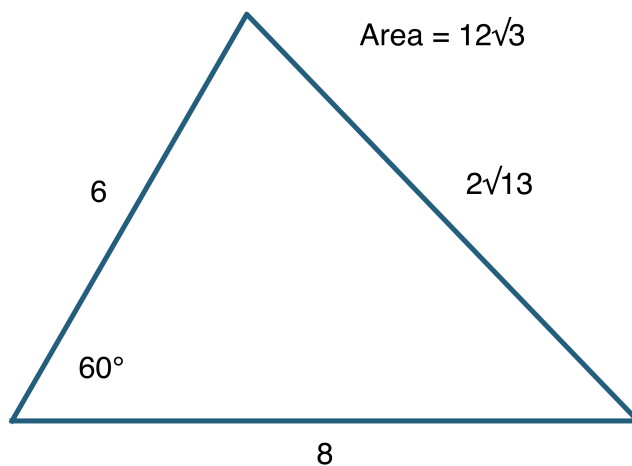
Label opposite, adjacent and hypotenuse first: 3, 4 and 5 here

Bisected equilateral triangle: sides 1, $\sqrt{3}$, 2



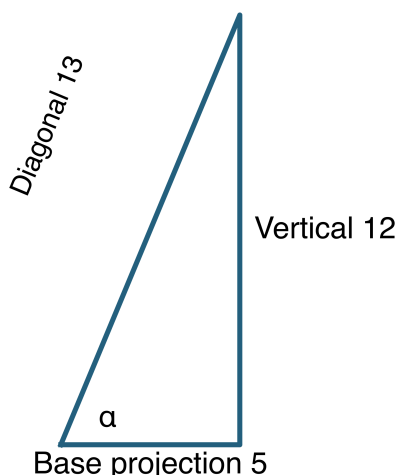
$\sin 30^\circ = 1/2$, $\cos 30^\circ = \sqrt{3}/2$, $\tan 60^\circ = \sqrt{3}$ from the 1, $\sqrt{3}$, 2 triangle

Two sides and their included angle



$\text{Area} = \frac{1}{2} \times 6 \times 8 \times \sin 60^\circ = 12\sqrt{3}$; cosine rule gives the third side $2\sqrt{13}$

Cuboid section: first $\sqrt{3^2+4^2}=5$, then $\sqrt{5^2+12^2}=13$



Base diagonal $\sqrt{3^2 + 4^2} = 5$, then space diagonal $\sqrt{5^2 + 12^2} = 13$; $\tan \alpha = 12/5$

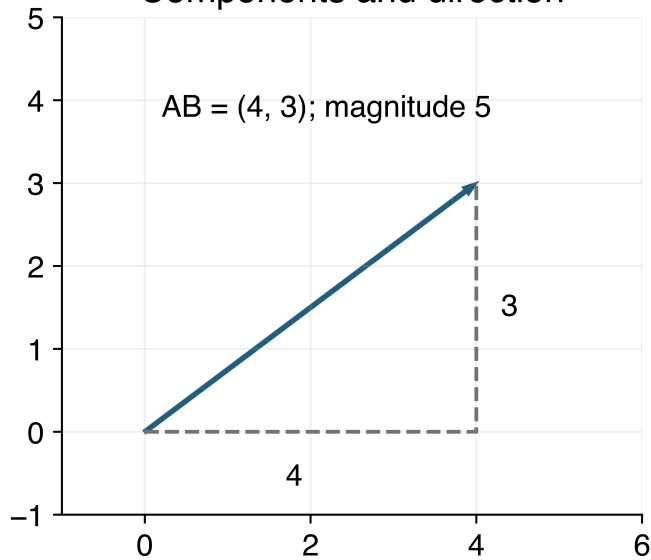
Vectors and coordinate geometry

Describe translations as 2D vectors (G24) – column notation, the top entry horizontal. Apply addition and subtraction of vectors, multiplication by a scalar, and diagrammatic and column representations (G25): with $\mathbf{a} = (4, 1)$ and $\mathbf{b} = (1, 3)$, $\mathbf{a} + \mathbf{b} = (5, 4)$ and $2\mathbf{a} - \mathbf{b} = (7, -1)$; from $A = (1, 2)$ to $B = (5, 5)$, $\overrightarrow{AB} = (4, 3)$ with length 5. Higher uses vectors to construct geometric arguments and proofs: a scalar multiple $\overrightarrow{MN} = \frac{1}{2}\overrightarrow{AB}$ proves MN parallel to AB and half its length (midpoints average position vectors); **collinear** points need a scalar-multiple relation AND a shared point.

Worked example: 2H June 2024 Q23, adapted. With $\overrightarrow{CD} = 6\mathbf{a} + \mathbf{b}$, $\overrightarrow{CE} = 14\mathbf{a} - 3\mathbf{b}$ and $\overrightarrow{CF} = 16\mathbf{a} - 4\mathbf{b}$: the displacement $\overrightarrow{DE} = (14\mathbf{a} - 3\mathbf{b}) - (6\mathbf{a} + \mathbf{b}) = 8\mathbf{a} - 4\mathbf{b}$, while $\overrightarrow{EF} = (16\mathbf{a} - 4\mathbf{b}) - (14\mathbf{a} - 3\mathbf{b}) = 2\mathbf{a} - \mathbf{b}$. So $\overrightarrow{DE} = 4\overrightarrow{EF}$ – parallel with a shared point E , so DEF is a straight line.

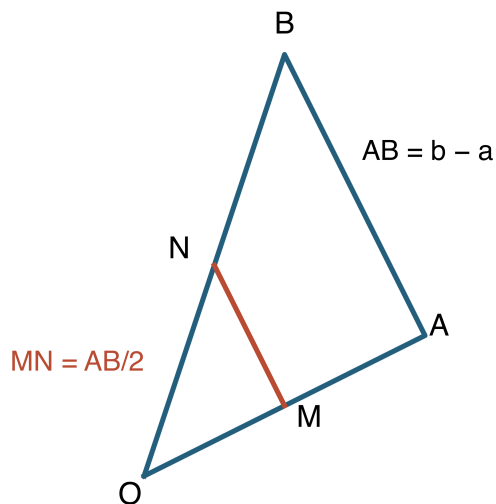
Solve geometrical problems on coordinate axes (G11): horizontal/vertical lengths by subtracting coordinates, diagonals by Pythagoras, midpoints by averaging – $A(1, 2)$, $B(5, 2)$, $C(5, 5)$, $D(1, 5)$ has $AB = 4$, $BC = 3$, diagonal $AC = 5$ and diagonals meeting at their common midpoint $(3, 3.5)$: a rectangle.

Components and direction



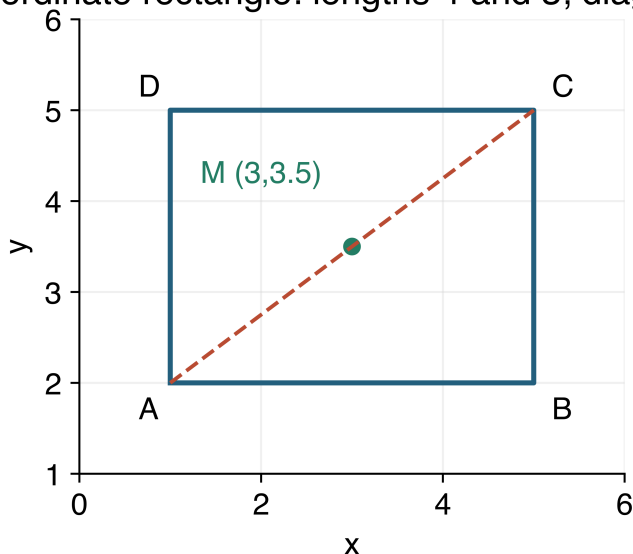
$$AB = (4, 3) \text{ has magnitude } \sqrt{4^2 + 3^2} = 5$$

Midpoint segment: half length, parallel direction



$$MN = \frac{1}{2}b - \frac{1}{2}a = \frac{1}{2}(b - a) = \frac{1}{2}AB: \text{half the length and parallel to } AB$$

Coordinate rectangle: lengths 4 and 3; diagonal 5



Sides 4 and 3 give diagonal 5; the midpoint of AC is (3, 3.5)

Checklist

- Conventional terms and notations; triangle labelling; diagrams from written descriptions (G1).
- Ruler and compass constructions incl. 60° ; loci problems; shortest distance (G2).
- Angle facts at a point/line/vertically opposite; alternate and corresponding with reasons; polygon sums derived (G3).
- Quadrilateral and triangle properties derived and applied (G4); congruence criteria SSS SAS ASA RHS (G5; Q12 anchor); conjecture-derive-prove chains incl. Pythagoras and isosceles base angles (G6).
- Transformations incl. fractional and negative scale factors; change and invariance under combinations (G7–G8).
- Circle parts named (G9; Q1 anchor); circle theorems applied AND proved at Higher (G10; Q22 anchor).
- Coordinate-axis geometry (G11); solids' faces/edges/vertices (G12); plans and elevations (G13); units (G14); maps, scale drawings, bearings (G15).
- Areas and prism/cylinder volumes (G16); circle measures, spheres, cones, pyramids and frustums, exact π answers (G17); arcs and sectors (G18; Q12 anchor); the $k : k^2 : k^3$ ladder (G19; Q25 anchor).
- Sine rule with the ambiguous case: both angles when the opposite side is the shorter; sketch first (G22, H).

- Pythagoras and trig ratios in 2D and 3D (G20; Q18 anchor); exact trig values (G21; Q25 anchor); sine and cosine rules (G22); $\frac{1}{2}ab \sin C$ (G23).
- Translations as vectors; vector arithmetic; geometric proofs incl. collinearity (G24–G25; Q23 anchor).

Boundary notes: the enclosed Formulae Sheet carries the trapezium area, prism volume, circle circumference/area, Pythagoras, the three trig ratios, the sine and cosine rules and $\frac{1}{2}ab \sin C$ – it does NOT carry arc/sector formulae, cone/pyramid/sphere volumes, exact trig values or circle theorems; those must be known. Angle reasons must name the theorem, never letter shapes. Worked examples are adapted from the June 2024 Higher papers (see the meta’s assessment evidence) or original.

Probability

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AQA GCSE Mathematics 8300 topic 3.5 (specification Version 1.0, 12 September 2014, first exams 2017). Foundation rows P1–P9 run in three tiers; the additional foundation rows are empirical **unbiased** samples tending toward theoretical distributions (P5), tree diagrams as a systematic device (P6) and the probability of **independent** and **DEPENDENT** combined events with their underlying assumptions (P8); the Higher-only row is conditional probability through **expected frequencies** with **two-way tables**, tree diagrams and **Venn diagrams** (P9). The June 2024 Higher set supplies verified anchors below.

Must-know for topic 3.5:

Family	Formulae
scale	$0 \leq P \leq 1$; exhaustive outcomes : $\sum P = 1$; mutually exclusive complement $P(A') = 1 - P(A)$
empirical	relative frequency $= \frac{\text{times occurred}}{\text{trials}}$; expected count $= n \times p$
combining	independent: $P(A \cap B) = P(A) \times P(B)$; dependent: adjust the second-stage denominator
conditioning	$P(A B) = \frac{P(A \cap B)}{P(B)}$, $P(B) > 0$

Experiments, frequency and expectation

Record, describe and analyse the frequency of outcomes of probability experiments using tables and **frequency tree** (P1) – probabilities written as fractions, decimals or percentages. Apply ideas of **randomness**, **fairness** and **equally likely** events to calculate EXPECTED outcomes of multiple future experiments (P2): for a stated probability p , the expected count in n future trials is np – a long-run average, never a guarantee. A spinner landing red 18 times in 60 spins estimates $P(\text{red}) = \frac{18}{60} = 0.3$, so 200 future spins expect $0.3 \times 200 = 60$ reds; 48 or 54 heads in 100 fair tosses is possible even though 50 is expected.

Worked example: 3H June 2024 Q10, adapted. The probability of rain on a June day in a town is 0.38. Of 150 days, the expected number of dry days: $1 - 0.38 = 0.62$, then $0.62 \times 150 = 93$.

Understand that EMPIRICAL unbiased samples tend toward the theoretical distribution with increasing **sample size** (P5) – 6 heads in 10 tosses estimates 0.6; 502 in 1000 estimates 0.502 – larger samples are more stable, never exact.

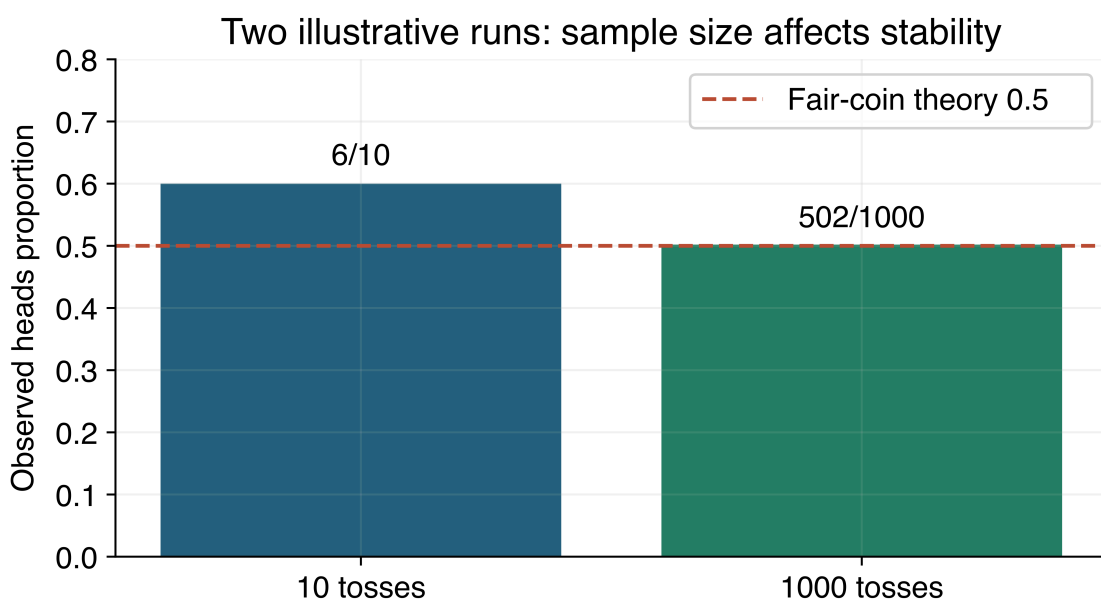
Worked example: 2H June 2024 Q7, adapted. Four students flip a coin different numbers of times; one gets 52 heads in 200 flips. The best estimate of $P(\text{head})$ is $\frac{52}{200}$ – it comes from the largest number of flips.

The probability scale, fairness and exhaustive sums

Relate relative expected frequencies to **theoretical probability** using appropriate language and the 0-to-1 **probability scale** (P3); apply the property that the probabilities of an exhaustive set of (mutually exclusive) outcomes or events sum to 1 (P4). If red, blue and green are exhaustive with probabilities 0.3 and 0.45, the third is $1 - 0.75 = 0.25$.

Worked example: 2H June 2024 Q4(b), adapted. A claimed probability list for a spinner adds to 1.1 – impossible: the probabilities of an exhaustive set of mutually exclusive outcomes sum to 1 (0.1 too much).

Worked example: 3H June 2024 Q16, adapted. Two dice are rolled 72 times; a total of 7 occurs 14 times. Theoretical = $\frac{6}{36} = \frac{1}{6}$ (expected $72 \times \frac{1}{6} = 12$); relative frequency = $\frac{14}{72} = \frac{7}{36}$, which exceeds $\frac{1}{6}$. This result alone does not establish **bias**: ordinary sampling variation can explain a difference from the expected frequency.



A small run can be far from 0.5; a large run settles near it: 0.6 against 0.502

Systematic listing and possibility spaces

Enumerate sets and combinations of sets SYSTEMATICALLY, using tables, grids and Venn diagrams, including tree diagrams at additional foundation (P6). Construct theoretical **possibility spaces** for single and combined experiments with equally likely outcomes and use them to calculate theoretical probabilities (P7): a coin and a die give 12 equally likely ordered outcomes, heads-with-even is $\frac{3}{12} = \frac{1}{4}$; two dice give 36 ordered pairs – total seven has $\frac{6}{36} = \frac{1}{6}$ and at-least-eleven has $\frac{3}{36} = \frac{1}{12}$. State whether order matters, and why the listed outcomes are equally likely, before dividing counts.

Worked example: 3H June 2024 Q22, adapted. A code uses four different digits from five: $5 \times 4 \times 3 \times 2 = 120$ possible codes – count systematically down each stage, never $5^4 = 625$, which allows repeated digits. (The count happens to equal $5! = 120$ only because, once four digits are chosen, the fifth is forced.)

In a class of 30, 18 cycle, 12 swim and 7 do both: the Venn regions fill overlap-first – cycling only 11, swimming only 5, at least one = 23, **neither** 7; the two-way table checks back to 18, 12, 30. **Union** , **intersection** and complement translate the words both, either/at least one, only and neither into counted regions; shade the correct region.

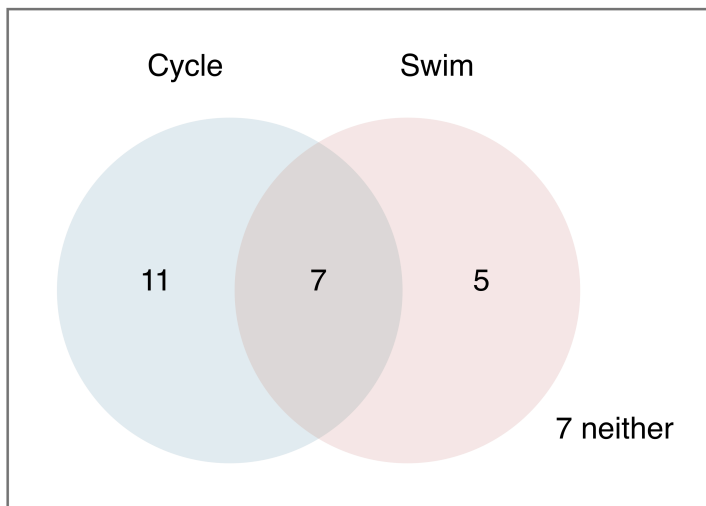
Worked example: 2H June 2024 Q9, adapted. On a Venn diagram of two sets, “both A and B” shades the INTERSECTION – the overlap lens only.

36 equally likely cells; six sums equal seven

Second die	6	7	8	9	10	11	12
	5	6	7	8	9	10	11
	4	5	6	7	8	9	10
	3	4	5	6	7	8	9
	2	3	4	5	6	7	8
	1	2	3	4	5	6	7
		1	2	3	4	5	6
		First die					

36 equally likely outcomes; six sum to 7, so $P(7) = 6/36 = 1/6$

30 students: overlap first, then only and neither



Fill the overlap first: $11 + 7 + 5 = 23$ do at least one, so 7 do neither

Combined events and tree diagrams

Calculate the probability of independent and dependent combined events, including using **tree diagrams** and other representations, and know the underlying assumptions (P8) – including WHEN TO ADD and when to multiply: multiply along a path (successive stages), add across disjoint paths (mutually exclusive routes). A bag with 3 red and 2 blue counters: **with replacement** $P(RR) = \frac{3}{5} \times \frac{3}{5} = \frac{9}{25}$; **without replacement** $P(RR) = \frac{3}{5} \times \frac{2}{4} = \frac{3}{10}$ and $P(\text{one of each}) = \frac{3}{5} \times \frac{2}{4} + \frac{2}{5} \times \frac{3}{4} = \frac{3}{5}$. Independent events keep every stage's denominator fixed – justify independence from the context (replacement, separate trials) before using a bare product; two disjoint events with positive probability are NOT independent.

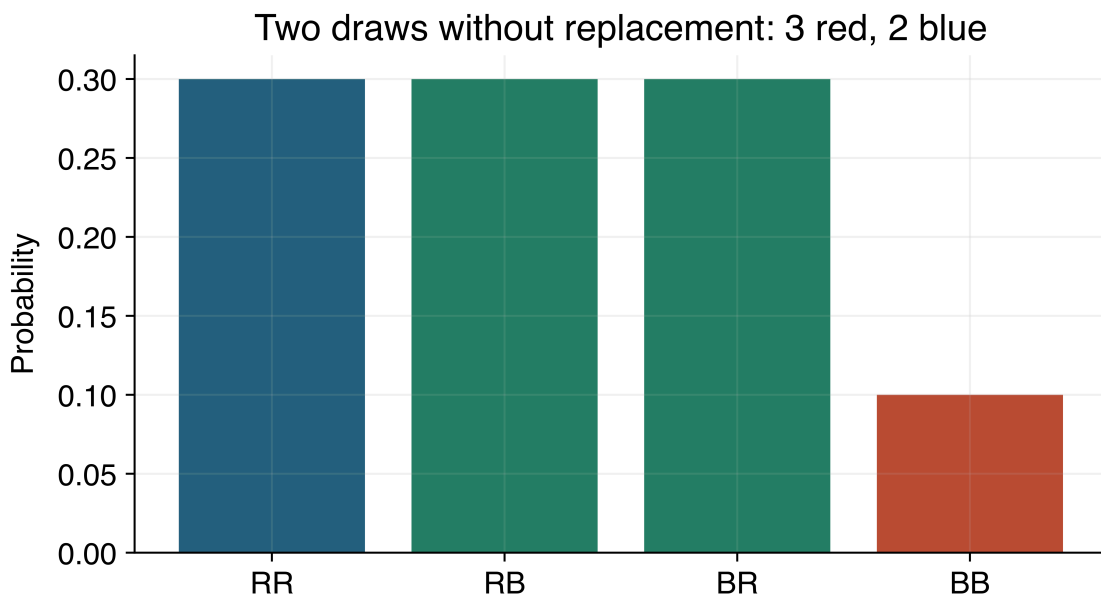
Worked example: 3H June 2024 Q12(b), adapted. A machine fails on a given day with probability 0.05, independently each day. $P(\text{at least one failure in two days}) = 1 - 0.95 \times 0.95 = 1 - 0.9025 = 0.0975$ – the complement route beats summing the three failing paths.

Conditional probability

Higher (P9): calculate and interpret **conditional probability** through expected frequencies with two-way tables, tree diagrams and Venn diagrams – $P(A | B) = \frac{P(A \cap B)}{P(B)}$ with $P(B) > 0$: conditioning RESTRICTS the denominator to the given group. From the class of 30 (18 cycle, 12 swim, 7 both): $P(\text{swim} | \text{cycle}) = \frac{7}{18}$ while $P(\text{cycle} | \text{swim}) = \frac{7}{12}$ – reversing the condition changes the denominator, and neither

equals the unconditional $\frac{12}{30}$. With expected frequencies: a condition affecting 20% of 1000 people, a test positive for 90% of those with it and 10% of those without, gives $180 + 80 = 260$ positives, of which $\frac{180}{260} = \frac{9}{13} \approx 0.692$ have the condition – not 90%. Given-positive and positive-given-condition are different questions.

Worked example: Higher tier. The 3-red-2-blue bag, no replacement, REVERSED: given the SECOND counter is red, what is the probability the first was blue? $P(\text{second R}) = \frac{3}{5} \times \frac{2}{4} + \frac{2}{5} \times \frac{3}{4} = \frac{12}{20}$; $P(\text{blue then red}) = \frac{2}{5} \times \frac{3}{4} = \frac{6}{20}$; so $P(\text{first B} \mid \text{second R}) = \frac{6}{20} \div \frac{12}{20} = \frac{1}{2}$. Counting agrees: 12 of the 20 ordered pairs end in red; 6 of those start blue.



Without replacement: $P(RR) = \frac{3}{5} \times \frac{2}{4} = 0.3$, $P(BB) = \frac{2}{5} \times \frac{1}{4} = 0.1$

Given positive: use 180 out of 260, not 180 out of 200

	Positive	Negative	Total
Condition	180	20	200
No condition	80	720	800
Total	260	740	1000

Given a positive test, use the positive column: 180 out of 260, not 180 out of 200

Checklist

- Frequency tables and frequency trees for experiments; probabilities as fractions, decimals, percentages (P1).
- Randomness, fairness, equally likely events; expected outcomes of multiple future experiments (P2; Q10 anchor: $0.62 \times 150 = 93$).
- Relative frequency vs theoretical probability on the 0-to-1 scale (P3; Q16 anchor: $\frac{14}{72}$ vs $\frac{1}{6}$).
- Exhaustive mutually exclusive outcomes sum to 1; complements (P4; Q4b anchor).
- Empirical unbiased samples tend toward theory with larger samples (P5; Q7 anchor: $\frac{52}{200}$ from the most flips).
- Systematic enumeration with tables, grids, Venn diagrams, trees (P6; Q22 anchor: $5 \times 4 \times 3 \times 2 = 120$).
- Theoretical possibility spaces for single and combined equally likely experiments (P7; Q4a anchor: $\frac{2}{6}$).
- Independent and dependent combined events; multiply along paths, add disjoint routes; assumptions stated (P8; Q12b anchor: $1 - 0.95^2 = 0.0975$).
- Conditional probability via expected frequencies with two-way tables, trees, Venn diagrams (P9: $\frac{7}{18}$ vs $\frac{7}{12}$).

Boundary notes: nothing here appears on the Formulae Sheet; “expected” is a long-run average; justify independence from context before multiplying. Worked examples are adapted from the June 2024 Higher papers or original.

Statistics

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AQA GCSE Mathematics 8300 topic 3.6 (specification Version 1.0, 12 September 2014, first exams 2017). Rows S1–S6 run in three tiers; the additional foundation rows are inferring population properties from a sample while knowing the limitations of sampling (S1) and tables and line graphs for time series data (S2); the Higher-only row is **histograms** with equal and UNEQUAL **class intervals** and cumulative frequency graphs (S3), with **box plots**, **quartiles** and the inter-quartile range as the Higher addition to S4. The June 2024 Higher set supplies verified anchors below.

Must-know for topic 3.6:

Family	Formulae
centre	mean $= \frac{\sum fx}{\sum f}$ (midpoints for grouped data); median by cumulative count; mode / modal class
spread	range $= \max - \min$; interquartile range $= Q_3 - Q_1$
displays	pie angle $= \frac{f}{n} \times 360^\circ$; frequency density $= \frac{\text{frequency}}{\text{class width}}$
bivariate	correlation direction + strength; estimated line of best fit ; interpolate , extrapolate with care

Sampling and populations

Infer properties of populations or distributions from a sample, whilst knowing the limitations of sampling (S1, additional foundation): a **sample** is a subset; a **census** covers the whole **population**; random selection reduces selection bias but never removes sampling variability or non-response. Of a random sample of 80 students, 24 walk: proportion $\frac{24}{80} = 0.3$, applied to a school of 600 suggests about 180 walkers – an estimate, not a count. A convenience sample from a sports club overrepresents active students; a voluntary online poll misses non-respondents however many responses arrive. Apply statistics to describe a population (S5): state what the sample supports and what might make extrapolating unreliable. Know the terms **primary data**, **secondary data**, **discrete data** and **continuous data** (number of siblings is discrete; travel time is continuous even when recorded to whole minutes).

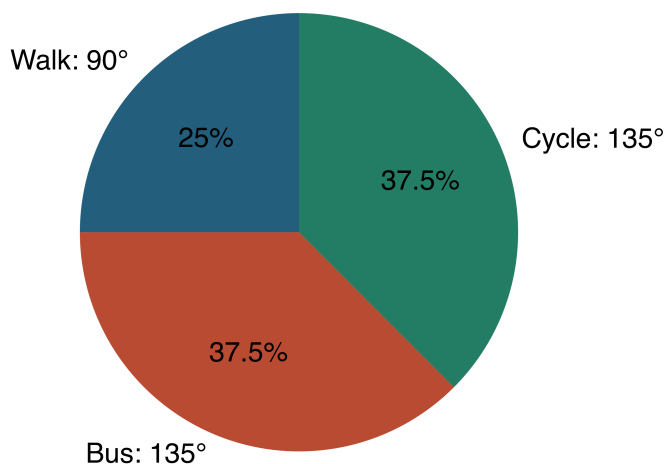
Tables, charts and time series

Interpret and construct tables, charts and diagrams (S2): frequency tables, bar charts, **pie charts** and pictograms for **categorical data**, vertical line charts for ungrouped discrete numerical data – and, at additional foundation, tables and line graphs for **time series** data – knowing their appropriate use, including

choosing suitable statistical diagrams. A pie-chart sector angle is $\frac{f}{n} \times 360^\circ$: of 40 students with 10 walking, 15 bussing and 15 cycling, the angles are 90° , 135° , 135° . Time series plot in order and join point-to-point; a **scatter graph** never joins in observation order.

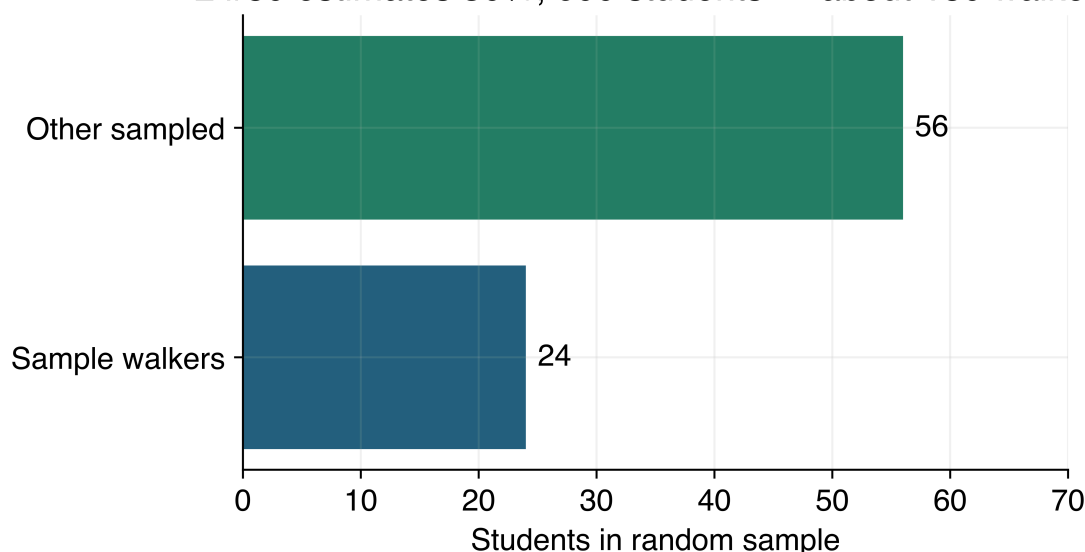
Worked example: 3H June 2024 Q8, adapted. A table of six values has mean $(8+9+9+6+9+10) \div 6 = 8.5$; a pie chart's 162° sector is $\frac{162}{360} \times 100 = 45\%$. A question combining "four times the mean" with the percentage: $4 \times 8.5 + 45 = 34 + 45 = 79$.

40 students: frequencies determine sector angles



Each student is $360^\circ \div 40 = 9^\circ$: 10 walkers make 90° , 15 cyclists 135°

24/80 estimates 30%; 600 students → about 180 walkers

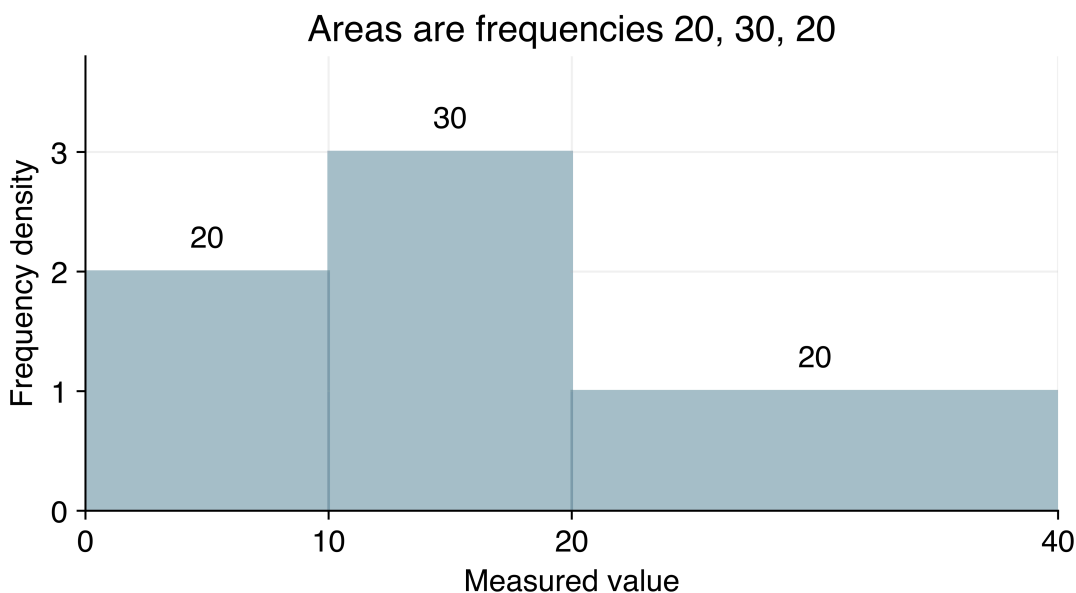


24 out of 80 is 30%, so about $0.3 \times 600 = 180$ walkers in the school

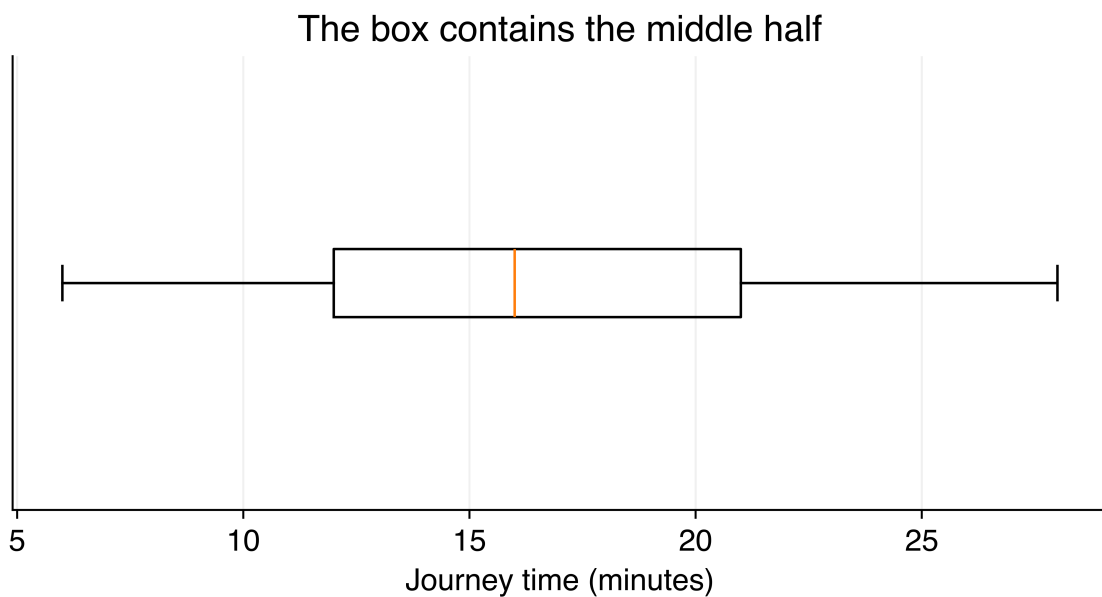
Histograms and cumulative frequency

Higher (S3): construct and interpret diagrams for grouped discrete and continuous data – histograms with equal and unequal class intervals and **cumulative frequency** graphs. Histogram AREA is frequency, so height (density) is frequency $\div$ **class width** : classes $0 \leq x < 10$, $10 \leq x < 20$, $20 \leq x < 40$ with frequencies 20, 30, 20 have densities 2, 3, 1 – the wide last bar is the shortest yet still holds 20. A cumulative-frequency graph plots each upper class boundary against the running total, starting at zero: for those classes the points are (0, 0), (10, 20), (20, 50), (40, 70); the median reads at 35, giving $10 + \frac{35-20}{30} \times 10 = 15$. Values within classes are interpolation estimates.

Worked example: Higher tier. From the same graph ($n = 70$): Q_1 reads at $17.5 - 10 \times \frac{17.5}{20} = 8.75$; Q_3 reads at $52.5 - 20 + \frac{52.5-50}{20} \times 20 = 22.5$; so $IQR = 22.5 - 8.75 = 13.75$. 'How many above 25?' reads at 25: $50 + \frac{5}{20} \times 20 = 55$, so $70 - 55 = 15$ – the graph answers counting questions the table does not.



Area = frequency: $10 \times 2 = 20$, $10 \times 3 = 30$, $20 \times 1 = 20$



The box holds the middle half: quartiles 12 and 21 minutes, IQR 9, median 16

Averages, spread and box plots

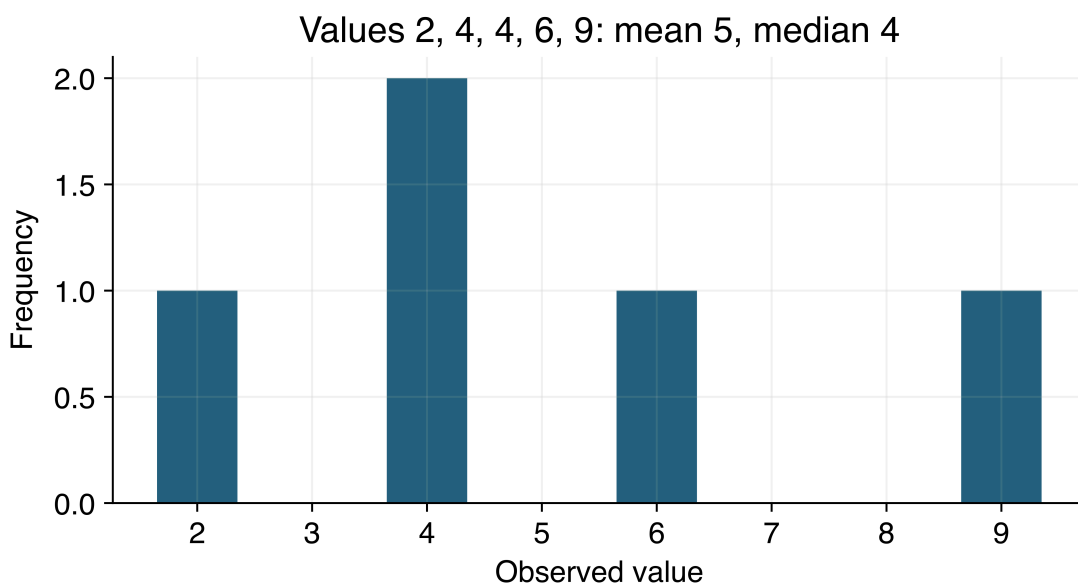
Interpret, analyse and compare distributions of data sets from univariate empirical distributions through appropriate graphical representation and appropriate measures of **central tendency** – median, mean, mode and modal class – and spread: range, including consideration of **outliers** (S4). From a frequency table, mean is $\frac{\sum fx}{\sum f}$: values 1, 2, 3 with frequencies 2, 5, 3 give $\frac{2+10+9}{10} = 2.1$; grouped data use midpoints, so the result is an estimate. The unusual value in 2, 4, 4, 6, 24 lifts the mean to 8 while the median stays 4.

Worked example: 2H June 2024 Q13, adapted. A table of lengths: 15 values of 7.2, 18 of 7.6 and 7 of 8 give $\sum fx = 108 + 136.8 + 56 = 300.8$; mean = $300.8 \div 40 = 7.52$ – keep the products exact (approximated fx values lose the method mark).

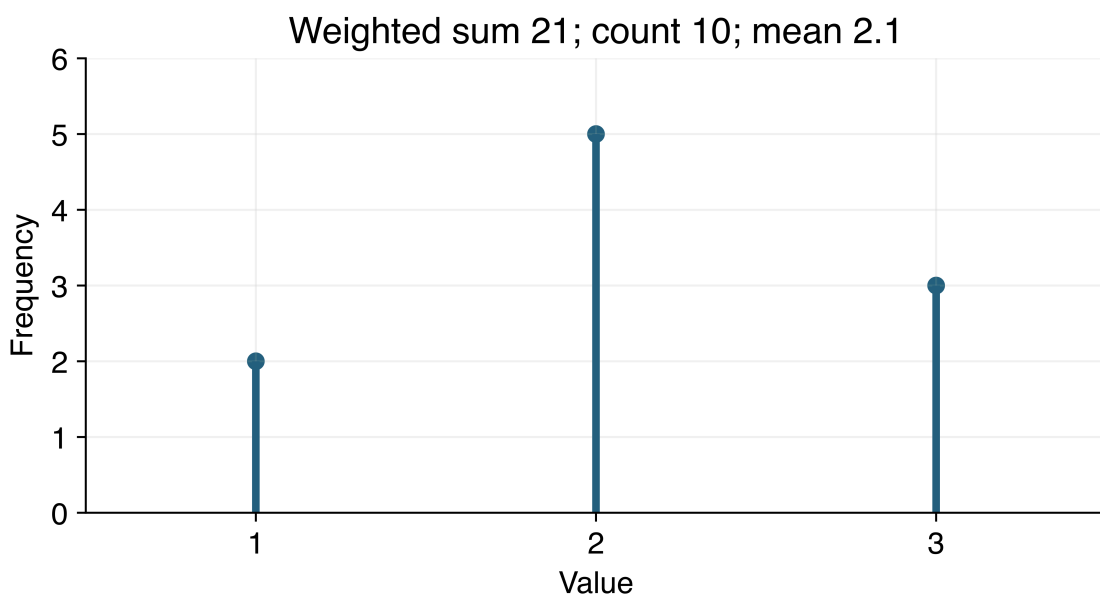
Higher adds box plots, quartiles and the inter-quartile range: a box plot marks minimum, Q_1 , median, Q_3 , maximum; $IQR = Q_3 - Q_1$ describes the middle half. Compare the MEDIAN for a typical value and the IQR for consistency, in context, with units.

Worked example: 2H June 2024 Q16, adapted. A box plot with shortest value 2.4 cm, lower quartile 4 cm, median 6 cm, upper quartile 7 cm and longest value 9 cm – five points, one rectangle with whiskers.

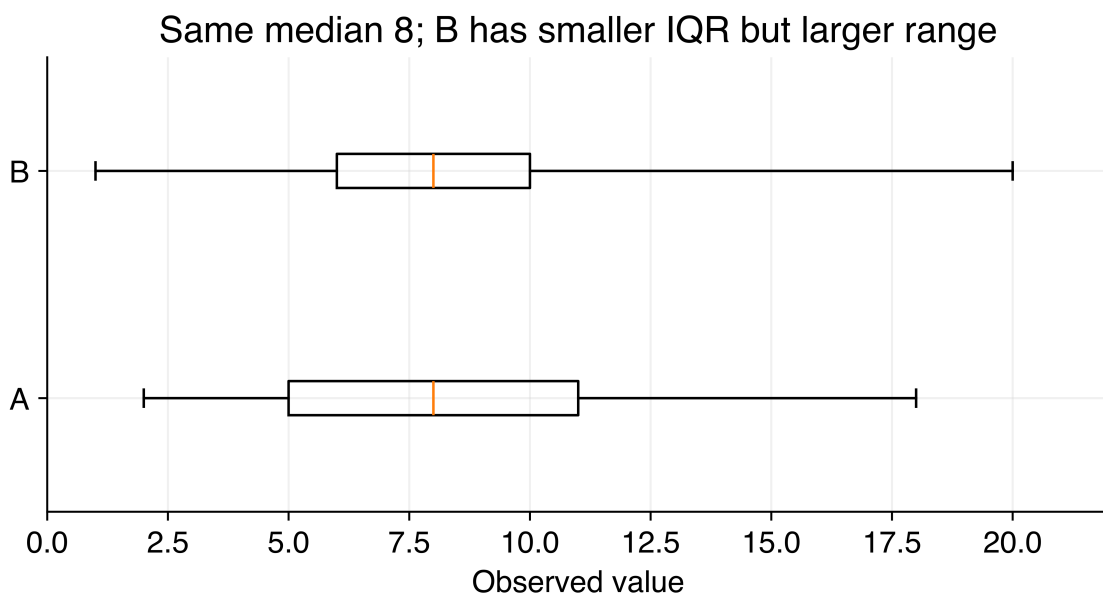
Worked example: 1H June 2024 Q16, adapted. Two clubs' ages: "the swimming club's median age is lower, so its average age is lower" AND "the swimming club's inter-quartile range is higher, so its ages are less consistent" – a comparison needs BOTH an average and a spread statement.



For 2, 4, 4, 6, 9: mean = $25 \div 5 = 5$, median = 4



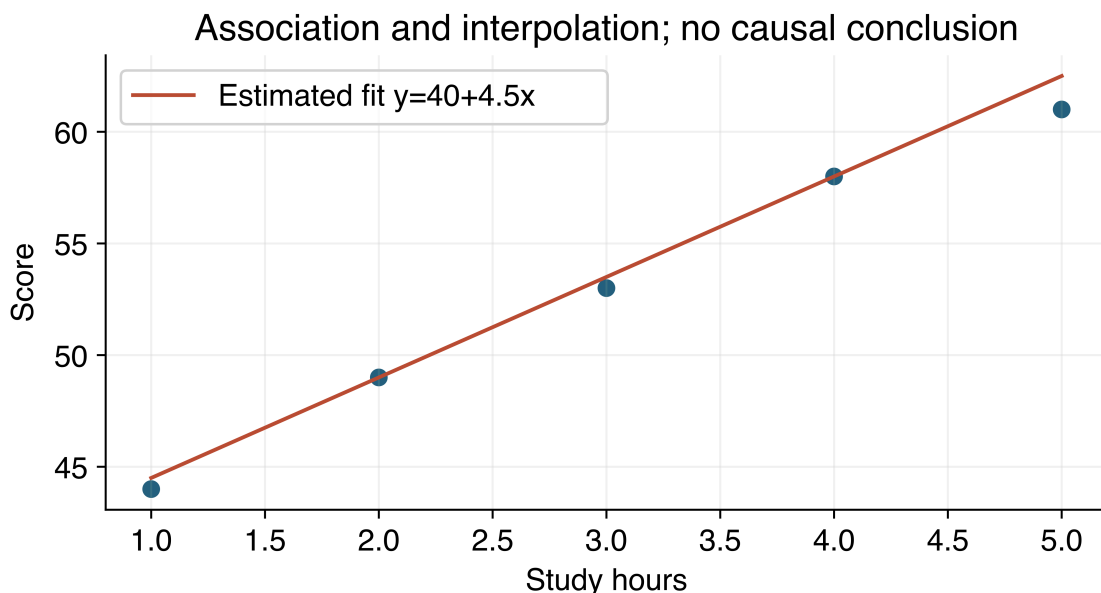
$$\text{Mean} = (1 \times 2 + 2 \times 5 + 3 \times 3) \div 10 = 21 \div 10 = 2.1$$



Same median 8; B has the smaller IQR (4 against 6) but the larger range

Scatter graphs and correlation

Use and interpret scatter graphs of bivariate data (S6): recognise positive, negative, no, weak and strong correlation; additional foundation knows that correlation does NOT indicate causation; draw estimated lines of best fit through the middle of the pattern (balancing points, not necessarily through the origin) and make predictions; interpolate and extrapolate apparent trends whilst knowing the dangers. Study hours 1–5 against scores 44, 49, 53, 58, 61 with line $\hat{y} = 40 + 4.5x$: predicting at $x = 3.6$ interpolates ($\hat{y} = 56.2$); at $x = 10$ the line gives 85, but gains may flatten – state the observed input range with every prediction.



Use the line to interpolate inside the data, for example $4.5 \rightarrow 60$; it shows association, not cause

Checklist

- Samples vs census; inference to populations with limitations stated; primary/secondary, discrete/continuous data (S1, S5).
- Frequency tables, bar/pie/pictogram, vertical line charts, time-series tables and line graphs; choosing suitable diagrams (S2; Q8 anchor).
- Histograms with equal and unequal widths (area = frequency, density = $f \div \text{width}$) and cumulative frequency graphs at boundaries (S3).
- Mean from tables with exact products; median/mode/modal class; range and outliers; box plots, quartiles, IQR; median-and-IQR comparisons (S4; Q13, Q16, 1H Q16 anchors).

- Scatter graphs: correlation types and strength; correlation is not causation; estimated best-fit lines; interpolation safe, extrapolation dangerous (S6).

Boundary notes: nothing for this topic appears on the enclosed Formulae Sheet – the mean, range, pie angle and density relationships must be known. Comparisons state an average AND a spread with context and units. Worked examples are adapted from the June 2024 Higher papers (see the meta’s assessment evidence) or original.