

Probability

AQA · GCSE · Mathematics

AQA GCSE Mathematics 8300 topic 3.5 (specification Version 1.0, 12 September 2014, first exams 2017). Foundation rows P1–P9 run in three tiers; the additional foundation rows are empirical unbiased samples tending toward theoretical distributions (P5), tree diagrams as a systematic device (P6) and the probability of independent and DEPENDENT combined events with their underlying assumptions (P8); the Higher-only row is conditional probability through expected frequencies with two-way tables, tree diagrams and Venn diagrams (P9). The June 2024 Higher set supplies verified anchors below.

Must-know for topic 3.5:

Family	Formulae
scale	$0 \leq P \leq 1$; exhaustive mutually exclusive outcomes: $\sum P = 1$; complement $P(A') = 1 - P(A)$
empirical	relative frequency $= \frac{\text{times occurred}}{\text{trials}}$; expected count $= n \times p$
combining	independent: $P(A \cap B) = P(A) \times P(B)$; dependent: adjust the second-stage denominator
conditioning	$P(A B) = \frac{P(A \cap B)}{P(B)}$, $P(B) > 0$

Experiments, frequency and expectation

Record, describe and analyse the frequency of outcomes of probability experiments using tables and **frequency tree** (P1) – probabilities written as fractions, decimals or percentages. Apply ideas of randomness, fairness and equally likely events to calculate EXPECTED outcomes of multiple future experiments (P2): for a stated probability p , the expected count in n future trials is np – a long-run average, never a guarantee. A spinner landing red 18 times in 60 spins estimates $P(\text{red}) = \frac{18}{60} = 0.3$, so 200 future spins expect $0.3 \times 200 = 60$ reds; 48 or 54 heads in 100 fair tosses is possible even though 50 is expected.

Worked example (3H June 2024 Q10, adapted). The probability of rain on a June day in a town is 0.38. Of 150 days, the expected number of dry days: $1 - 0.38 = 0.62$, then $0.62 \times 150 = 93$.

Understand that EMPIRICAL unbiased samples tend toward the theoretical distribution with increasing sample size (P5) – 6 heads in 10 tosses estimates 0.6; 502 in 1000 estimates 0.502; larger samples are more stable but never forced to agree exactly.

Worked example (2H June 2024 Q7, adapted). Four students flip a coin different numbers of times; one gets 52 heads in 200 flips. The best estimate of $P(\text{head})$ is $\frac{52}{200}$

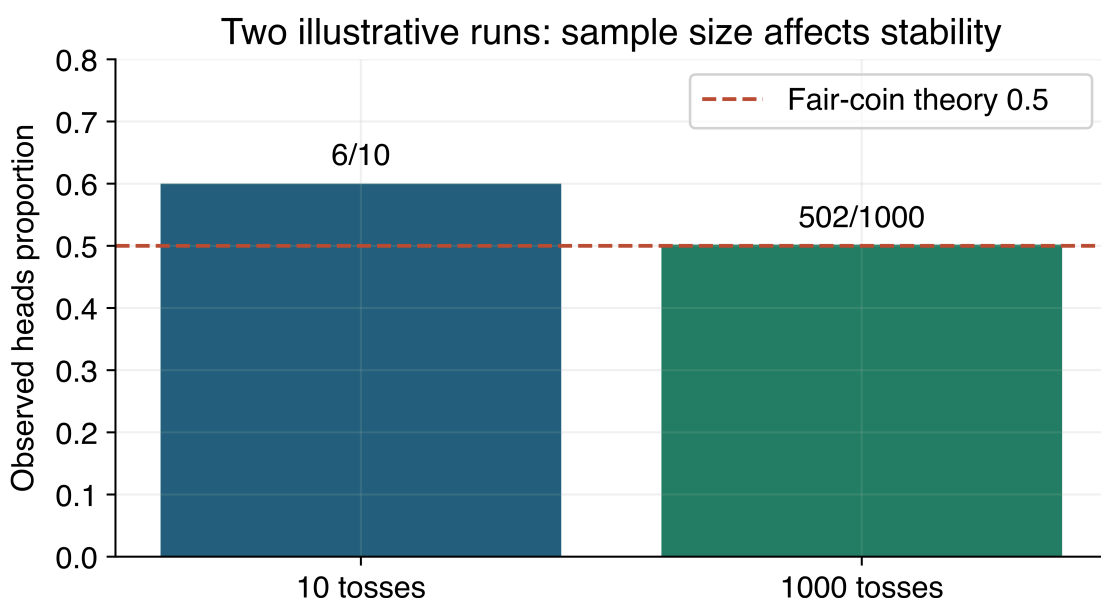
– it comes from the largest number of flips.

The probability scale, fairness and exhaustive sums

Relate relative expected frequencies to theoretical probability using appropriate language and the 0-to-1 **probability scale** (P3); apply the property that the probabilities of an exhaustive set of (mutually exclusive) outcomes or events sum to 1 (P4). If red, blue and green are exhaustive with probabilities 0.3 and 0.45, the third is $1 - 0.75 = 0.25$.

Worked example (2H June 2024 Q4(b), adapted). A claimed probability list for a spinner adds to 1.1 – impossible: the probabilities of an exhaustive set of mutually exclusive outcomes sum to 1 (0.1 too much).

Worked example (3H June 2024 Q16, adapted). Two dice are rolled 72 times; one named total occurs 14 times. Theoretical = $\frac{1}{6}$ (expected $72 \times \frac{1}{6} = 12$); relative frequency = $\frac{14}{72} = \frac{7}{36}$, which exceeds $\frac{1}{6}$. This result alone does not establish bias: ordinary sampling variation can explain a difference from the expected frequency.



Original native-lesson illustration; labels belong to its worked example.

Systematic listing and possibility spaces

Enumerate sets and combinations of sets **SYSTEMATICALLY**, using tables, grids and Venn diagrams, including tree diagrams at additional foundation (P6). Construct theoretical **possibility space** s for single and combined experiments with equally likely outcomes and use them to calculate theoretical probabilities (P7): a coin and a die give 12 equally likely ordered outcomes, heads-with-even is $\frac{3}{12} = \frac{1}{4}$; two dice

give 36 ordered pairs – total seven has $\frac{6}{36} = \frac{1}{6}$ and at-least-eleven has $\frac{3}{36} = \frac{1}{12}$. State whether order matters, and why the listed outcomes are equally likely, before dividing counts.

Worked example (3H June 2024 Q22, adapted). A code uses four different digits from five: $5 \times 4 \times 3 \times 2 = 120$ possible codes – count systematically down each stage, never 5^4 (repetition) nor $5!$ (one digit too many).

In a class of 30, 18 cycle, 12 swim and 7 do both: the Venn regions fill overlap-first – cycling only 11, swimming only 5, at least one = 23, **neither** 7; the two-way table rows and columns then check back to 18, 12 and 30. Union, **intersection** and complement translate the words both, either/at least one, only and neither into counted regions; shade the correct region.

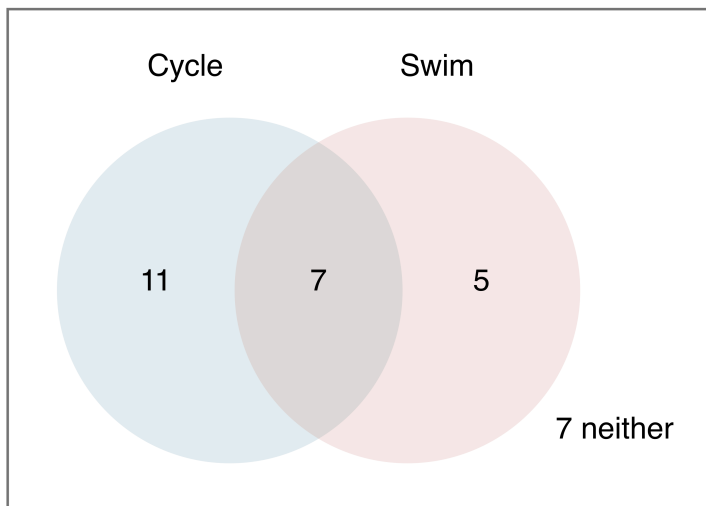
Worked example (2H June 2024 Q9, adapted). On a Venn diagram of two sets, “both A and B” shades the INTERSECTION – the overlap lens only.

36 equally likely cells; six sums equal seven

Second die	6	7	8	9	10	11	12
	5	6	7	8	9	10	11
	4	5	6	7	8	9	10
	3	4	5	6	7	8	9
	2	3	4	5	6	7	8
	1	2	3	4	5	6	7
		1	2	3	4	5	6
		First die					

Original native-lesson illustration; labels belong to its worked example.

30 students: overlap first, then only and neither



Original native-lesson illustration; labels belong to its worked example.

Combined events and tree diagrams

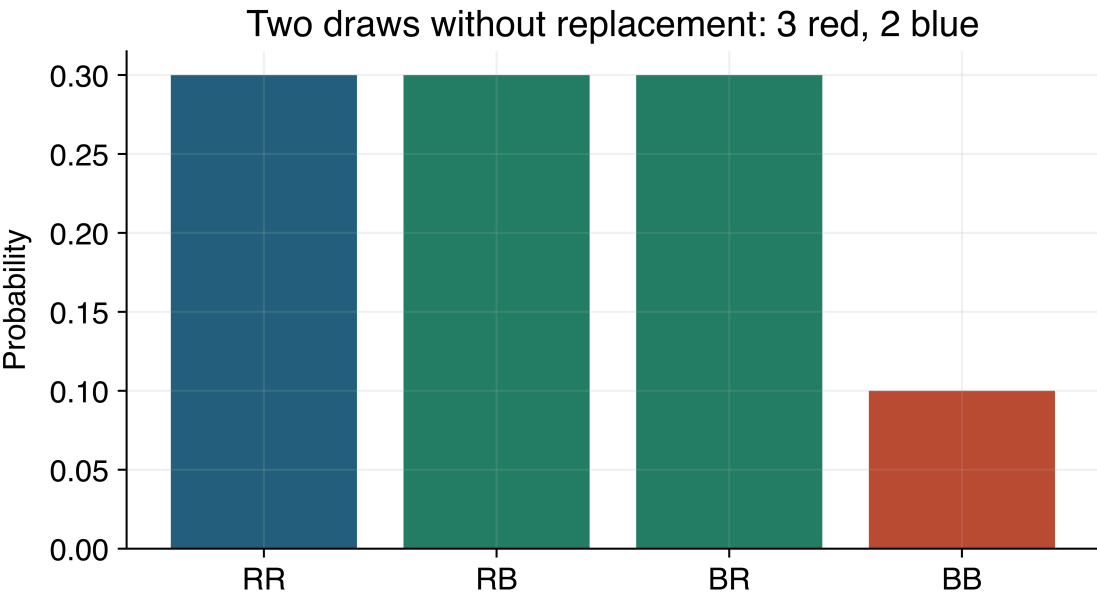
Calculate the probability of independent and dependent combined events, including using **tree diagram** s and other representations, and know the underlying assumptions (P8) – including WHEN TO ADD and when to multiply: multiply along a path (successive stages), add across disjoint paths (mutually exclusive routes). A bag with 3 red and 2 blue counters: with replacement $P(RR) = \frac{3}{5} \times \frac{3}{5} = \frac{9}{25}$; without replacement $P(RR) = \frac{3}{5} \times \frac{2}{4} = \frac{3}{10}$ and $P(\text{one of each}) = \frac{3}{5} \times \frac{2}{4} + \frac{2}{5} \times \frac{3}{4} = \frac{3}{5}$. Independent events keep every stage's denominator fixed – justify independence from the context (replacement, separate trials) before using a bare product; two disjoint events with positive probability are NOT independent.

Worked example (3H June 2024 Q12(b), adapted). A machine fails on a given day with probability 0.05, independently each day. $P(\text{at least one failure in two days}) = 1 - 0.95 \times 0.95 = 1 - 0.9025 = 0.0975$ – the complement route beats summing the three failing paths.

Conditional probability

Higher (P9): calculate and interpret **conditional probability** through expected frequencies with two-way tables, tree diagrams and Venn diagrams – $P(A | B) = \frac{P(A \cap B)}{P(B)}$ with $P(B) > 0$: conditioning RESTRICTS the denominator to the given group. From the class of 30 (18 cycle, 12 swim, 7 both): $P(\text{swim} | \text{cycle}) = \frac{7}{18}$ while $P(\text{cycle} | \text{swim}) = \frac{7}{12}$ – reversing the condition changes the denominator, and neither equals the unconditional $\frac{12}{30}$. With expected frequencies: a condition affecting 20% of

1000 people, a test positive for 90% of those with it and 10% of those without, gives $180 + 80 = 260$ positives, of which $\frac{180}{260} = \frac{9}{13} \approx 0.692$ have the condition – not 90%. Given-positive and positive-given-condition are different questions.



Original native-lesson illustration; labels belong to its worked example.

Given positive: use 180 out of 260, not 180 out of 200

	Positive	Negative	Total
Condition	180	20	200
No condition	80	720	800
Total	260	740	1000

Original native-lesson illustration; labels belong to its worked example.

Checklist

- Frequency tables and frequency trees for experiments; probabilities as fractions, decimals, percentages (P1).
- Randomness, fairness, equally likely events; expected outcomes of multiple future experiments (P2; Q10 anchor: $0.62 \times 150 = 93$).
- Relative frequency vs theoretical probability on the 0-to-1 scale (P3; Q16 anchor: $\frac{14}{72}$ vs $\frac{1}{6}$).
- Exhaustive mutually exclusive outcomes sum to 1; complements (P4; Q4b anchor).
- Empirical unbiased samples tend toward theory with larger samples (P5; Q7 anchor: $\frac{52}{200}$ from the most flips).
- Systematic enumeration with tables, grids, Venn diagrams, trees (P6; Q22 anchor: $5 \times 4 \times 3 \times 2 = 120$).
- Theoretical possibility spaces for single and combined equally likely experiments (P7; Q4a anchor: $\frac{2}{6}$).
- Independent and dependent combined events; multiply along paths, add disjoint routes; assumptions stated (P8; Q12b anchor: $1 - 0.95^2 = 0.0975$).
- Conditional probability via expected frequencies with two-way tables, trees, Venn diagrams (P9: $\frac{7}{18}$ vs $\frac{7}{12}$).

Boundary notes: nothing for this topic appears on the enclosed Formulae Sheet; “expected” is a long-run average, not a promise; independence must be justified from context before multiplying. Worked examples are adapted from the June 2024 Higher papers (see the meta’s assessment evidence) or original.