

Geometry and measures

AQA · GCSE · Mathematics

AQA GCSE Mathematics 8300 topic 3.4 (specification Version 1.0, 12 September 2014, first exams 2017). Foundation rows G1–G25 run in three tiers; the Higher-only additions are negative enlargement scale factors (G7), describing change and invariance under combinations of transformations (G8), proving the standard circle theorems (G10), constructing plans and elevations (G13), surface area and volume of spheres, pyramids, cones and composite solids incl. frustums (G17), the $k : k^2 : k^3$ similarity ladder (G19), right and general triangles in 3D (G20), the sine and cosine rules (G22), $\frac{1}{2}ab \sin C$ (G23) and geometric vector proofs (G25). The June 2024 Higher set supplies verified anchors below.

Must-know for topic 3.4:

Family	Formulae
angle sums	triangle 180° ; polygon interior $(n - 2) \times 180^\circ$; exterior turn $\frac{360^\circ}{n}$
circles	$C = 2\pi r = \pi d$; $A = \pi r^2$; arc $= \frac{\theta}{360^\circ} \cdot 2\pi r$; sector $= \frac{\theta}{360^\circ} \cdot \pi r^2$
plane/solid	triangle $\frac{1}{2}bh$; parallelogram bh ; trapezium $\frac{1}{2}(a + b)h$; prism $A_{\text{cross}}L$; cylinder $\pi r^2 h$; cone $\frac{1}{3}\pi r^2 h$; sphere $\frac{4}{3}\pi r^3$, $4\pi r^2$
triangles	$a^2 + b^2 = c^2$; $\sin \theta = \frac{\text{opp}}{\text{hyp}}$, $\cos \theta = \frac{\text{adj}}{\text{hyp}}$, $\tan \theta = \frac{\text{opp}}{\text{adj}}$; sine rule $\frac{a}{\sin A} = \frac{b}{\sin B}$; cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$; area $\frac{1}{2}ab \sin C$
similarity	lengths k : areas k^2 : volumes k^3

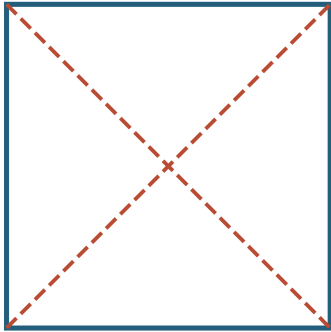
Terms, angle facts and polygons

Use conventional terms and notations – points, lines, vertices, edges, planes, parallel lines, perpendicular lines, right angles, polygons, **regular polygon** (all sides and all angles equal) and polygons with reflection and/or rotation symmetries (G1); label sides and angles of triangles in the standard way (AB a side, angle ABC at vertex B) and draw diagrams from written descriptions. **Corresponding angles** and alternate angles need parallel lines stated, never “Z-angles” (G3): angles at a point sum to 360° , on a line 180° , vertically opposite angles are equal; derive the triangle sum 180° (a line through a vertex parallel to the opposite base transfers the two base angles onto a straight line) and hence the polygon interior sum $(n - 2) \times 180^\circ$ – a pentagon splits into three triangles: 540° ; a regular hexagon has interior angles of 120° and exterior turns of 60° ; an exterior turn of 45° means 8 sides.

Derive and apply the properties and definitions of the special quadrilaterals (G4): a square has 4 equal sides, 4 right angles, 4 **line of symmetry** and rotational order 4; a non-square rectangle has opposite sides equal and equal bisecting diagonals; a rhombus has 4 equal sides and perpendicular bisecting diagonals; a parallelogram’s opposite sides are equal (a diagonal splits it into congruent triangles by ASA); a

kite has two pairs of adjacent equal sides; a trapezium has one pair of parallel sides.
Triangles: isosceles (two equal sides, equal base angles), equilateral, scalene, right-angled, acute, obtuse.

Square diagonals are symmetry axes



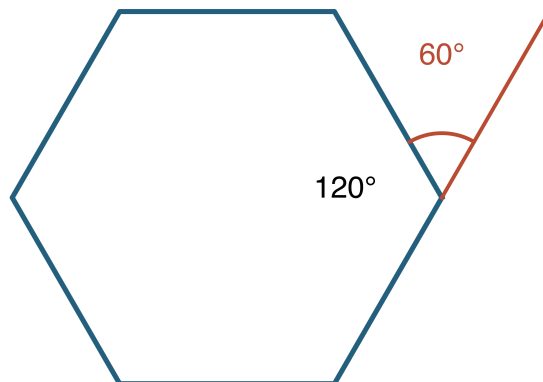
Square



Rectangle

Original native-lesson illustration; labels belong to its worked example.

Regular hexagon: interior 120° , exterior 60°



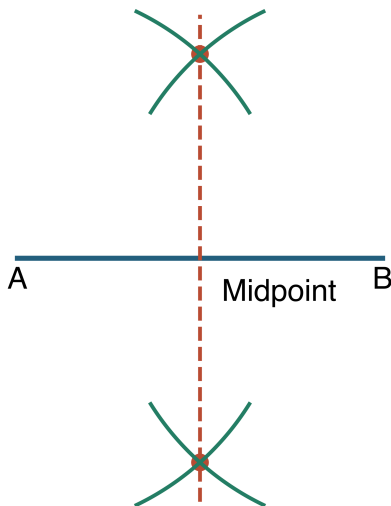
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Constructions, loci and transformations

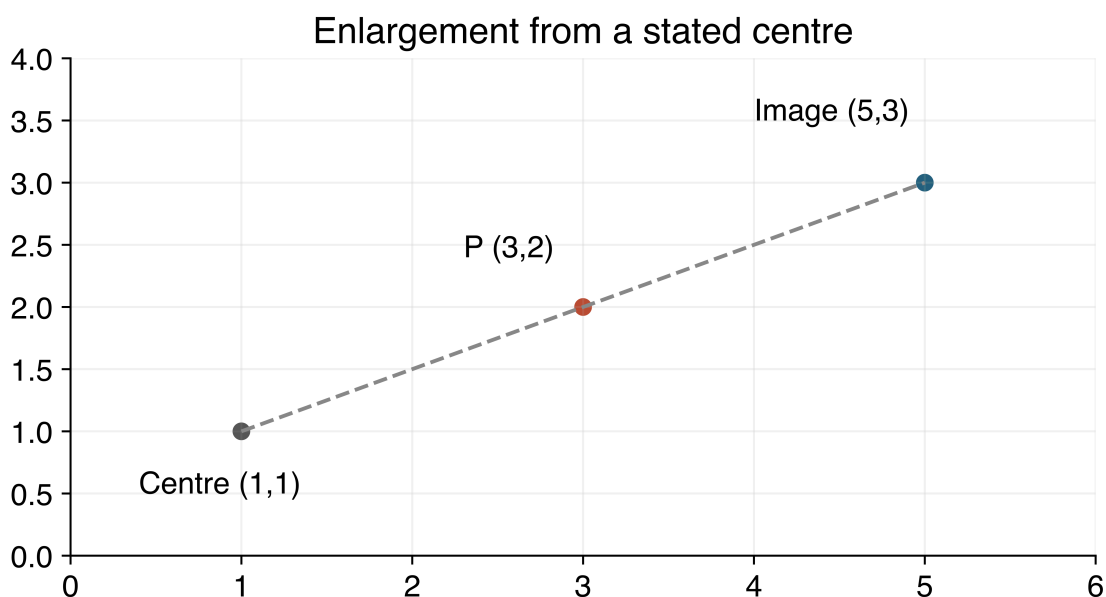
Use the standard ruler and compass **constructions** (additional foundation, G2): the **perpendicular bisector** of a segment (equal arcs from both endpoints, radius more than half the segment), the perpendicular to a line from or at a given point, and the bisector of a given angle; construct an angle of 60° from two equal-radius circles (an equilateral triangle). Use these to construct given figures and solve **locus** problems – points equidistant from two points lie on the perpendicular bisector; at fixed distance r from a point, on a circle; equidistant from two lines, on an angle bisector. The perpendicular distance from a point to a line is the shortest (any slanted route is a longer hypotenuse). Keep the construction arcs as evidence.

Identify, describe and construct congruent and similar shapes by rotation, reflection, translation and **enlargement** (G7), including on coordinate axes; additional foundation adds fractional scale factors; Higher adds negative ones. For centre $C = (1, 1)$, point $P = (3, 2)$ and factor k , the image is $C + k(P - C)$: $k = 2$ gives $(5, 3)$, $k = \frac{1}{2}$ gives $(2, 1.5)$, and the Higher $k = -2$ gives $(-3, -1)$, on the opposite side of the centre. Higher (G8) describes the changes and **invariant** properties achieved by combinations of rotations, reflections and translations: lengths and angles are preserved (isometries); two reflections in parallel lines $x = 0$ then $x = 2$ send $(3, 2)$ to $(-3, 2)$ then $(7, 2)$ – a translation by $(4, 0)$; in intersecting lines they give a rotation through twice the directed angle between the mirrors. Translations are written with column-vector notation.

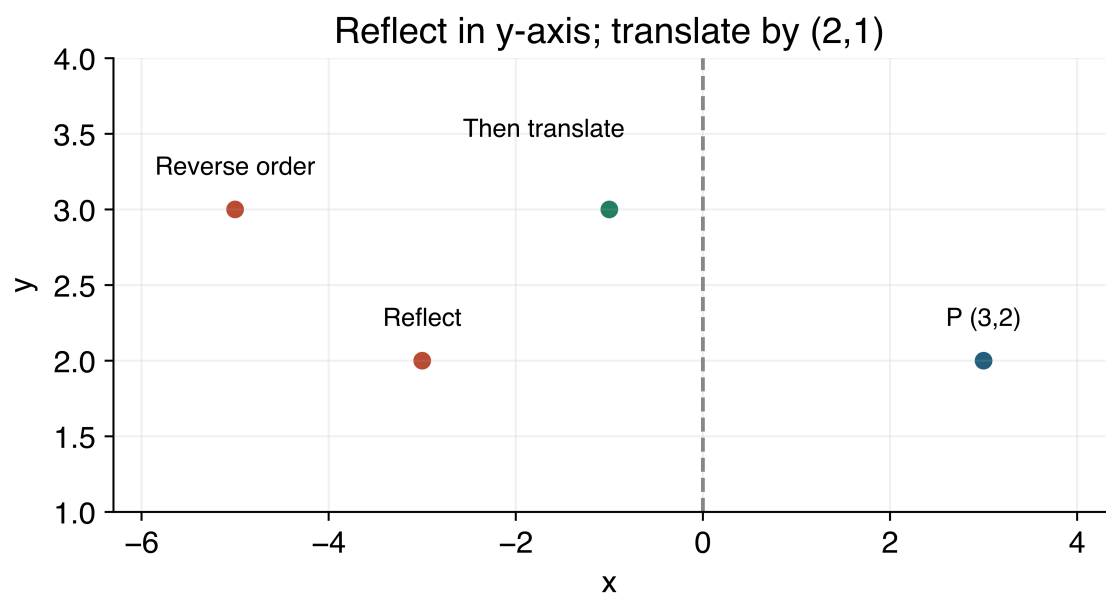
Equal compass radii: join the two arc intersections



Original native-lesson illustration; labels belong to its worked example.



Original native-lesson illustration; labels belong to its worked example.



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Congruence, similarity and circle theorems

Apply the basic **congruence** criteria for triangles – SSS, SAS, ASA, RHS (G5) – matching vertices in order; AAA gives similarity, not congruence, and SSA is not a criterion.

Worked example (2H June 2024 Q12, adapted). Two triangles share three equal corresponding sides. The one-word congruence reason: **SSS**.

Apply angle facts, triangle congruence, similarity and quadrilateral properties to conjecture and derive results, including Pythagoras' theorem and the equality of the base angles of an isosceles triangle, and use known results to obtain simple proofs (G6). For isosceles ABC with $AB = AC$ and D the **midpoint** of BC : triangles ABD and ACD agree in $AB = AC$, $BD = DC$ and shared AD , so SSS gives congruence and matching base angles $ABC = BCA$; the two angles at D are equal and sum to 180° , so each is 90° . Pythagoras follows from four congruent right triangles around a tilted square inside a square of side $a + b$: $(a + b)^2 = 4 \cdot \frac{ab}{2} + c^2$, hence $a^2 + b^2 = c^2$.

Identify and apply circle definitions and properties (G9): centre, radius, chord, diameter ($d = 2r$), **circumference**, with additional foundation adding tangent, arc, **sector** and **segment** – a sector is bounded by two radii and an arc; a segment lies between a chord and an arc. The perpendicular from the centre to a chord bisects it: a chord 3 cm from the centre of a radius-5 circle has length $2\sqrt{5^2 - 3^2} = 8$ cm.

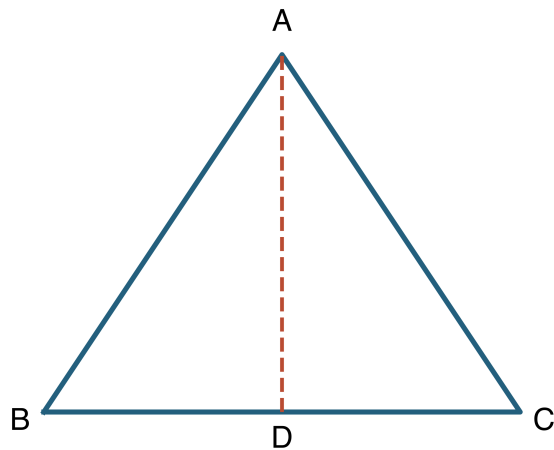
Worked example (2H June 2024 Q1, adapted). Name each shaded region from: arc, chord, sector, segment, tangent – the sliver between a chord and the arc is the segment; the pie-slice between two radii and their arc is the sector.

Higher (G10) applies and PROVES the standard circle theorems: the **angle at the centre** is twice the angle at the circumference on the same arc (prove with two isosceles triangles on equal radii: $\angle AOB = 360^\circ - (180^\circ - 2\alpha) - (180^\circ - 2\beta) = 2(\alpha + \beta)$); the angle in a semicircle is 90° ; angles in the same segment are equal; opposite angles of a **cyclic quadrilateral** sum to 180° ; the tangent at any point is perpendicular to the radius there; tangents from an external point are equal (RHS on the two contact triangles); the perpendicular from the centre to a chord bisects it; and the **alternate segment theorem** – the tangent-chord angle equals the angle in the opposite segment (a tangent-chord angle of 35° gives 35° in that segment).

Worked example (1H June 2024 Q22, adapted). Two tangents from P touch a circle at A and B ; angle $APB = 24^\circ$. Draw AO and BO : the quadrilateral $OAPB$ has two 90° contact angles, so the reflex angle at O is $360^\circ - 90^\circ - 90^\circ - 24^\circ = 156^\circ$, and the acute angle $x = \frac{156^\circ}{2} = 78^\circ$ (or: the angle at the centre on the same arc is $2x$, vertically opposite the reflex angle).

Congruence proves equal base angles

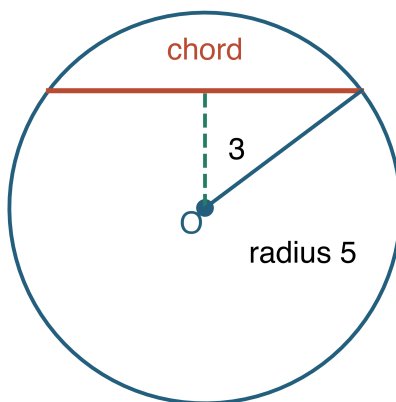
$$AB = AC$$



$$BD = DC; AD \text{ shared} \rightarrow \text{SSS}$$

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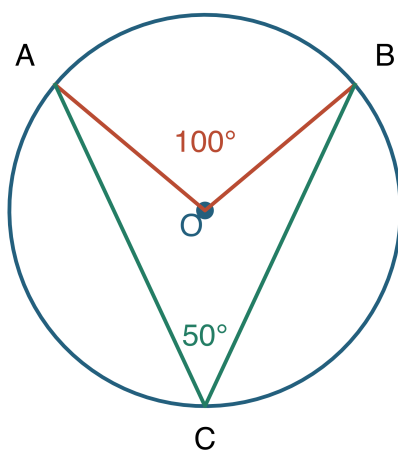
Circle of radius 5: distances use one consistent scale



Perpendicular bisects chord: half-length 4

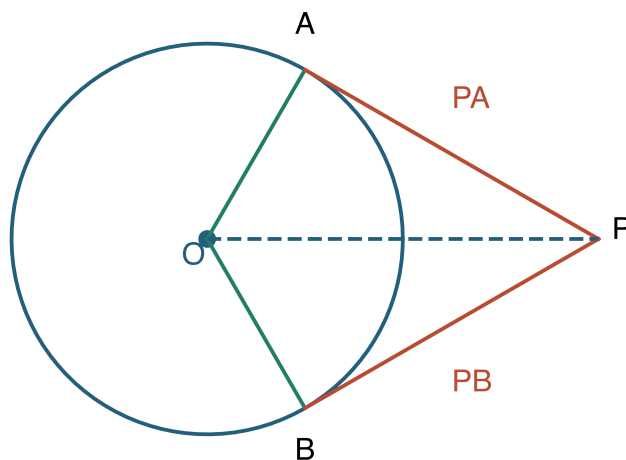
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Angles on the same upper arc: centre = twice circumference



Original native-lesson illustration; labels belong to its worked example.

Shared OP and equal radii $\rightarrow$ RHS $\rightarrow PA = PB$



Original native-lesson illustration; labels belong to its worked example.

Mensuration: perimeter, area and volume

Use standard units of measure and related concepts (G14). Identify the faces, surfaces, edges and vertices of cubes, cuboids, prisms, cylinders, pyramids, cones and spheres (G12); interpret – and at Higher construct – plans and elevations of 3D shapes (G13): for a $4 \times 3 \times 2$ cuboid the plan is 4×3 , the front **elevation** 4×2 , the side elevation 3×2 . Measure line segments and angles in geometric figures, including interpreting maps and scale drawings and use of **bearing** (G15): three-figure bearings measured clockwise from north – east 090° , and the reverse of 070° is $070^\circ + 180^\circ = 250^\circ$; on a 1 : 10000 map, 3 cm represents 300 m.

Know and apply the area formulae for triangles ($\frac{1}{2}b \times$ **perpendicular height**), parallelograms and trapezia, and volume of cuboids and other right prisms including cylinders (G16): a trapezium with parallel sides 6 and 10 and height 4 has area $\frac{1}{2}(6 + 10) \times 4 = 32 \text{ cm}^2$; a triangular prism with end area 12 cm^2 and length 10 cm has volume 120 cm^3 . Know the circle formulae (G17): $C = 2\pi r = \pi d$, $A = \pi r^2$; a radius-4 circle has circumference 8π and area 16π ; the semicircle on it has area 8π and perimeter $4\pi + 8$; an annulus with radii 5 and 3 has area $\pi(25 - 9) = 16\pi$. Additional foundation: surface area and volume of spheres, pyramids, cones and composite solids, incl. **frustum** – a cone with $r = 3$, $h = 4$ has slant 5, volume 12π , curved area 15π and total 24π ; a sphere with $r = 3$ has volume 36π and surface 36π ; a frustum from a 6, 8-cone (slant 10) minus a similar 3, 4-cone (slant 5) has volume $96\pi - 12\pi = 84\pi$. Solutions in terms of π may be asked for.

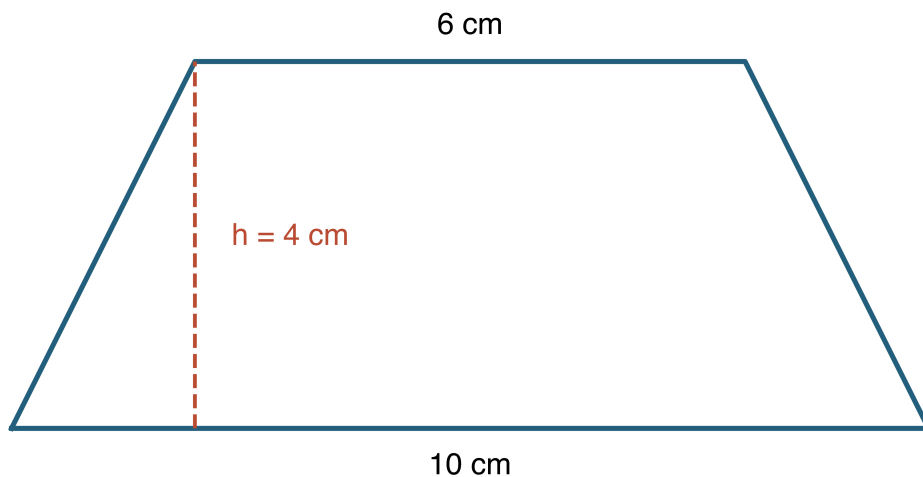
Calculate arc lengths, angles and areas of sectors (G18, additional foundation): at $r = 6$ and $\theta = 120^\circ$ the fraction is $\frac{1}{3}$, so the arc is 4π and the sector area is 12π ; a sector of radius 6 and area 9π has fraction $\frac{1}{4}$, so $\theta = 90^\circ$.

Worked example (1H June 2024 Q12, adapted). A sector of radius 12 cm has angle 60° : arc $= \frac{60}{360} \times 2 \times 12 \times \pi = 4\pi \text{ cm}$ – show the full-circle quantity and the fraction before multiplying.

Apply congruence and similarity, including the relationships between lengths, areas and volumes in similar figures (G19; Higher): the $k : k^2 : k^3$ ladder with **scale factor** k .

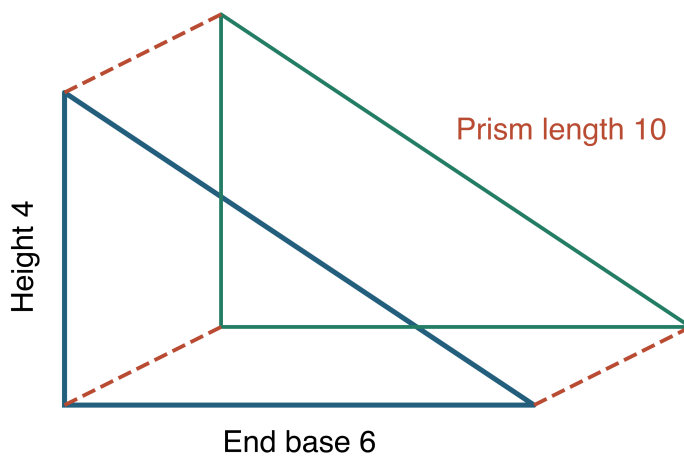
Worked example (2H June 2024 Q25, adapted). A right-angled triangular prism has edge ratio $a : b : c : d = 3 : 4 : 5 : 12$ and volume 1125 cm^3 . Write the edges as $3k, 4k, 5k, 12k$: volume $= \frac{1}{2}(3k)(4k)(12k) = 72k^3 = 1125$, so $k^3 = 15.625$ and $k = 2.5$. Total edge length $= 2(3 + 4 + 5)k + 3 \times 12k = 60k = 150 \text{ cm}$ (two of each cross-section edge, three length edges).

Trapezium area = average parallel sides $\times$ height



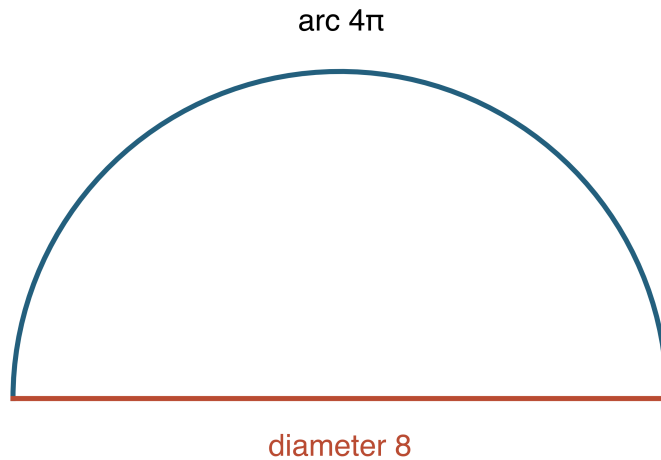
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Constant triangular end area 12 $\rightarrow$ volume 120



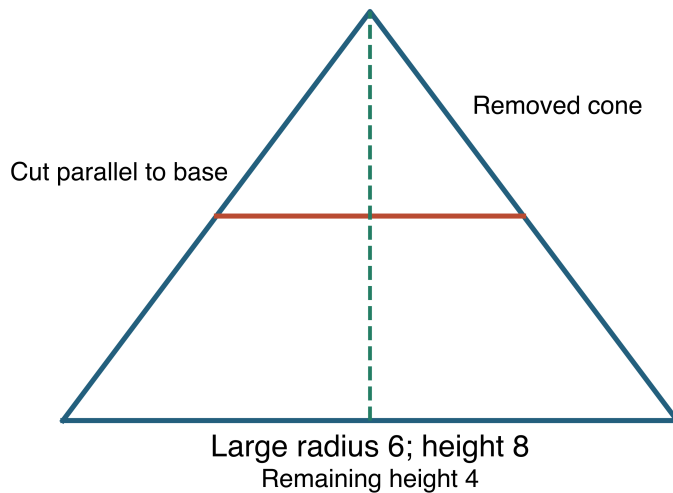
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Semicircle perimeter: curved arc + straight diameter



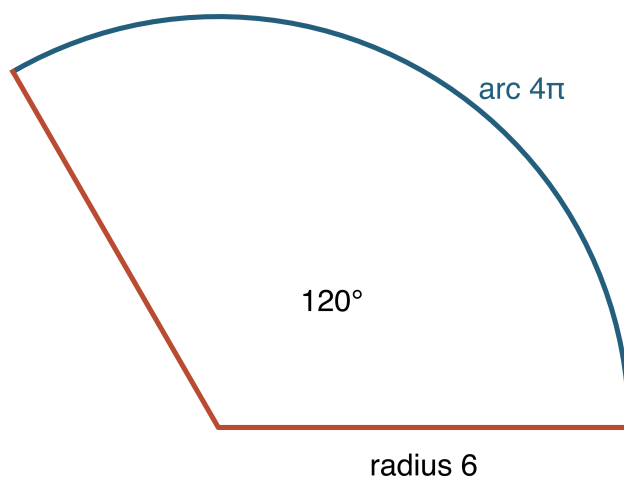
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Frustum volume = whole cone – removed similar cone



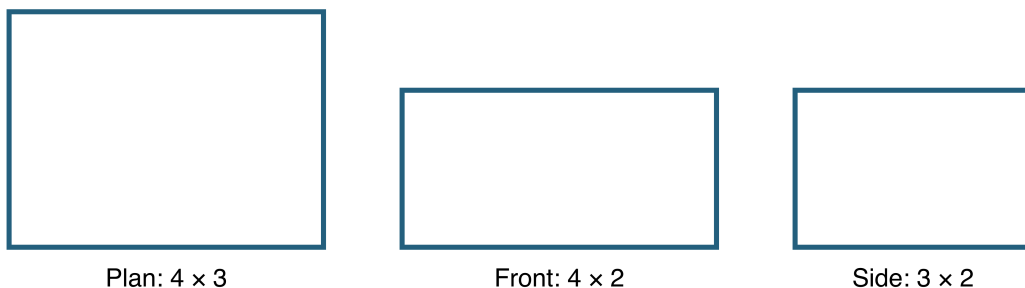
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One third of a turn: sector area 12π



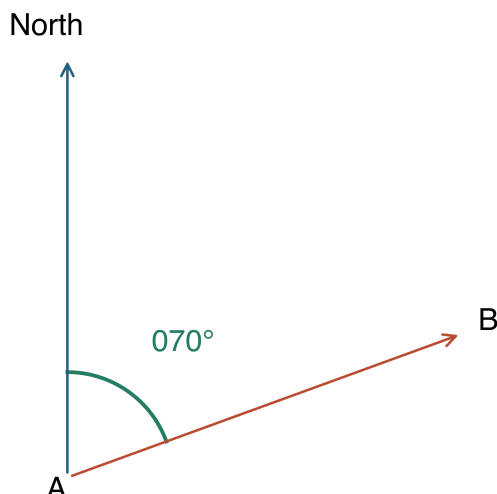
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Cuboid width 4, depth 3, height 2



Original native-lesson illustration; labels belong to its worked example.

Clockwise from north at A; reverse bearing at B is 250°



Original native-lesson illustration; labels belong to its worked example.

Pythagoras and trigonometry

Know Pythagoras' theorem $a^2 + b^2 = c^2$ and the trigonometric ratios (G20) – $\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$, $\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$, $\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$ – and apply them to find angles and lengths in right-angled triangles in 2D figures; Higher applies them (and, where possible, general-triangle rules) in 2D AND 3D. A 3-4-5 ladder footed 3 m out reaches $c = \sqrt{3^2 + 4^2} = 5$ m. In 3D, isolate the right triangle: a $3 \times 4 \times 12$ cuboid has base diagonal $\sqrt{9 + 16} = 5$, space diagonal $\sqrt{25 + 144} = 13$, and the space diagonal meets the base at $\tan \alpha = \frac{12}{5}$, so $\alpha \approx 67.4^\circ$.

Worked example (2H June 2024 Q18, adapted). In a right triangle the hypotenuse is 23.7 and x is the side **adjacent** to the marked angle 52° : $\cos 52^\circ = \frac{x}{23.7}$ (a sine-rule rearrangement $\frac{x}{\sin 38^\circ} = \frac{23.7}{\sin 90^\circ}$ also works), so $x = 23.7 \cos 52^\circ = 14.6$ (1 dp).

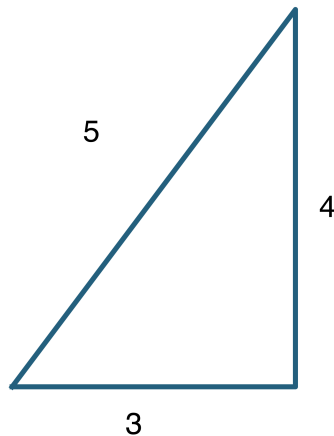
Know the **exact value** of $\sin \theta$ and $\cos \theta$ for $\theta = 0^\circ, 30^\circ, 45^\circ, 60^\circ, 90^\circ$ and of $\tan \theta$ for $0^\circ, 30^\circ, 45^\circ, 60^\circ$ (G21, additional foundation) – derive them from the bisected equilateral triangle $(1, \sqrt{3}, 2)$ and the right isosceles triangle $(1, 1, \sqrt{2})$: $\sin 30^\circ = \frac{1}{2}$, $\cos 30^\circ = \frac{\sqrt{3}}{2}$, $\tan 30^\circ = \frac{1}{\sqrt{3}}$, $\sin 45^\circ = \cos 45^\circ = \frac{\sqrt{2}}{2}$, $\tan 60^\circ = \sqrt{3}$; $\tan 90^\circ$ is undefined.

Worked example (1H June 2024 Q25, adapted). Show that $6 \sin 30^\circ + 2 \cos 30^\circ \times 4 \tan 30^\circ$ is an integer: $6 \times \frac{1}{2} + \sqrt{3} \times \frac{4}{\sqrt{3}} = 3 + 4 = 7$ – the surds cancel before any rounding.

Higher knows and applies the sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ and the cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$ (G22), and the area formula $A = \frac{1}{2}ab \sin C$ with the **included angle**

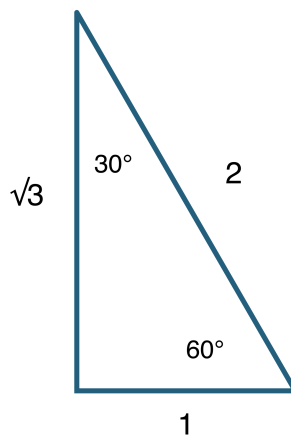
between a and b (G23). Sides 6 and 8 enclosing 60° give $c^2 = 36 + 64 - 96 \times \frac{1}{2} = 52$ so $c = 2\sqrt{13}$, and area $\frac{1}{2} \times 6 \times 8 \times \frac{\sqrt{3}}{2} = 12\sqrt{3}$. Sine rule: with $a = 4$ opposite $A = 30^\circ$ and $B = 45^\circ$, $b = \frac{4 \sin 45^\circ}{\sin 30^\circ} = 4\sqrt{2}$. An inverse sine may give an acute angle and an obtuse supplement – check the angle sum before choosing.

Label the known sides before choosing a rule



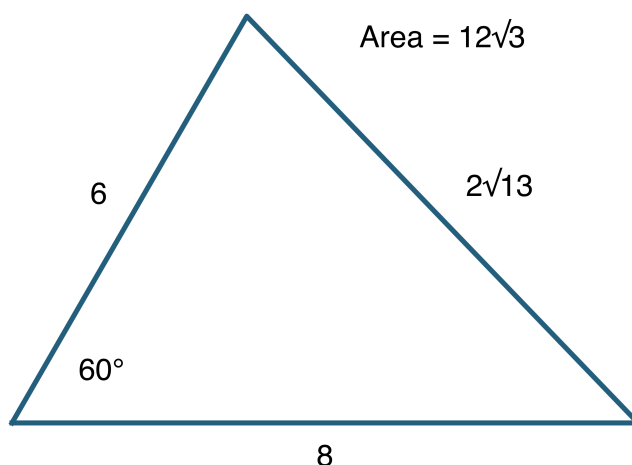
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Bisected equilateral triangle: sides 1, $\sqrt{3}$, 2



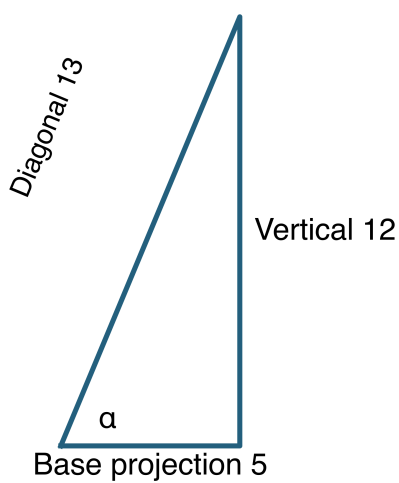
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Two sides and their included angle



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Cuboid section: first $\sqrt{3^2+4^2}=5$, then $\sqrt{5^2+12^2}=13$



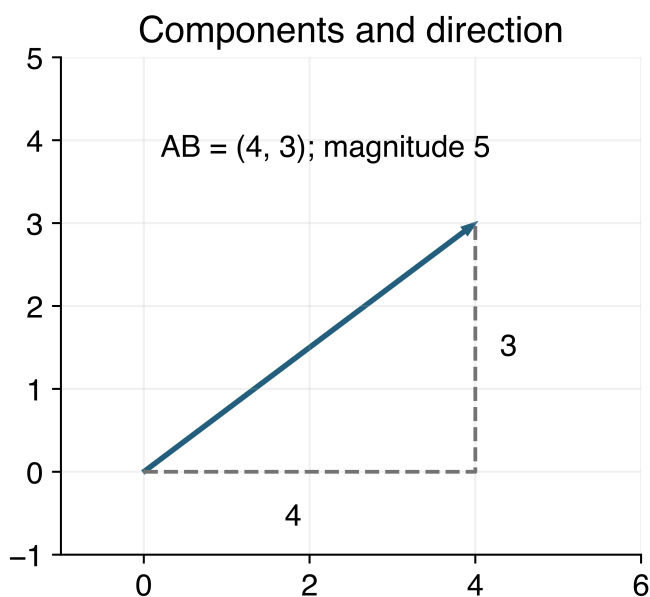
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Vectors and coordinate geometry

Describe translations as 2D vectors (G24) – column notation, the top entry horizontal. Apply addition and subtraction of vectors, multiplication by a scalar, and diagrammatic and column representations (G25): with $\mathbf{a} = (4, 1)$ and $\mathbf{b} = (1, 3)$, $\mathbf{a} + \mathbf{b} = (5, 4)$ and $2\mathbf{a} - \mathbf{b} = (7, -1)$; from $A = (1, 2)$ to $B = (5, 5)$, $\overrightarrow{AB} = (4, 3)$ with length 5. Higher uses vectors to construct geometric arguments and proofs: a scalar multiple $\overrightarrow{MN} = \frac{1}{2}\overrightarrow{AB}$ proves MN parallel to AB and half its length (midpoints average position vectors); **collinear** points need a scalar-multiple relation AND a shared point.

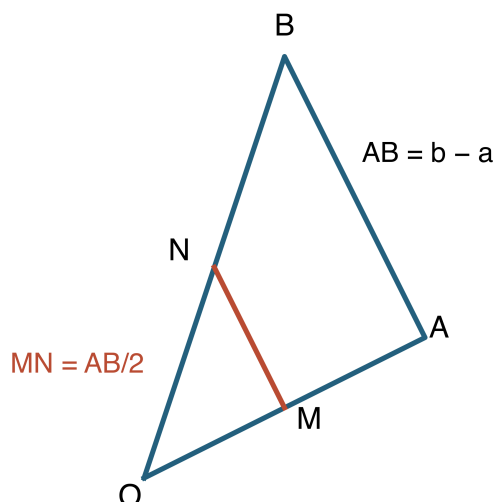
Worked example (2H June 2024 Q23, adapted). With $\overrightarrow{CD} = 6\mathbf{a} + \mathbf{b}$, $\overrightarrow{CE} = 14\mathbf{a} - 3\mathbf{b}$ and $\overrightarrow{CF} = 16\mathbf{a} - 4\mathbf{b}$: the displacement $\overrightarrow{DE} = (14\mathbf{a} - 3\mathbf{b}) - (6\mathbf{a} + \mathbf{b}) = 8\mathbf{a} - 4\mathbf{b}$, while $\overrightarrow{EF} = (16\mathbf{a} - 4\mathbf{b}) - (14\mathbf{a} - 3\mathbf{b}) = 2\mathbf{a} - \mathbf{b}$. So $\overrightarrow{DE} = 4\overrightarrow{EF}$ – parallel with a shared point E , so DEF is a straight line.

Solve geometrical problems on coordinate axes (G11): horizontal/vertical lengths by subtracting coordinates, diagonals by Pythagoras, midpoints by averaging – $A(1, 2)$, $B(5, 2)$, $C(5, 5)$, $D(1, 5)$ has $AB = 4$, $BC = 3$, diagonal $AC = 5$ and diagonals meeting at their common midpoint $(3, 3.5)$: a rectangle.



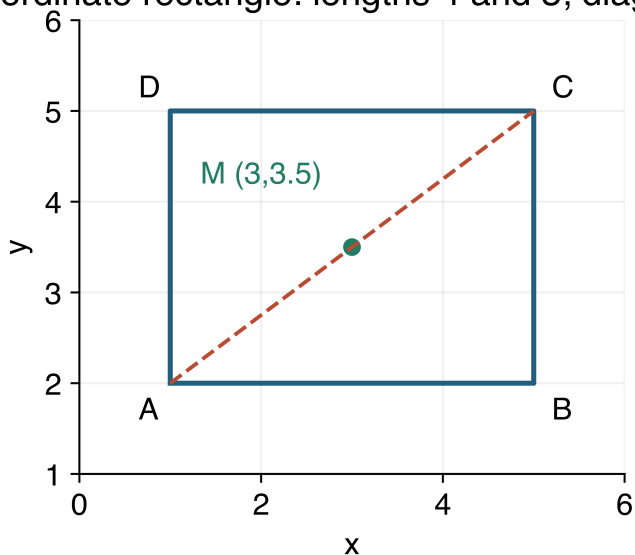
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Midpoint segment: half length, parallel direction



Original native-lesson illustration; labels belong to its worked example.

Coordinate rectangle: lengths 4 and 3; diagonal 5



Original native-lesson illustration; labels belong to its worked example.

Checklist

- Conventional terms and notations; triangle labelling; diagrams from written descriptions (G1).
- Ruler and compass constructions incl. 60° ; loci problems; shortest distance (G2).
- Angle facts at a point/line/vertically opposite; alternate and corresponding with reasons; polygon sums derived (G3).
- Quadrilateral and triangle properties derived and applied (G4); congruence criteria SSS SAS ASA RHS (G5; Q12 anchor); conjecture-derive-prove chains incl. Pythagoras and isosceles base angles (G6).
- Transformations incl. fractional and negative scale factors; change and invariance under combinations (G7–G8).
- Circle parts named (G9; Q1 anchor); circle theorems applied AND proved at Higher (G10; Q22 anchor).
- Coordinate-axis geometry (G11); solids' faces/edges/vertices (G12); plans and elevations (G13); units (G14); maps, scale drawings, bearings (G15).
- Areas and prism/cylinder volumes (G16); circle measures, spheres, cones, pyramids and frustums, exact π answers (G17); arcs and sectors (G18; Q12 anchor); the $k : k^2 : k^3$ ladder (G19; Q25 anchor).
- Pythagoras and trig ratios in 2D and 3D (G20; Q18 anchor); exact trig values (G21; Q25 anchor); sine and cosine rules (G22); $\frac{1}{2}ab \sin C$ (G23).
- Translations as vectors; vector arithmetic; geometric proofs incl. collinearity (G24–G25; Q23 anchor).

Boundary notes: the enclosed Formulae Sheet carries the trapezium area, prism volume, circle circumference/area, Pythagoras, the three trig ratios, the sine and cosine rules and $\frac{1}{2}ab \sin C$ – it does NOT carry arc/sector formulae, cone/pyramid/sphere volumes, exact trig values or circle theorems; those must be known. Angle reasons must name the theorem, never letter shapes. Worked examples are adapted from the June 2024 Higher papers (see the meta's assessment evidence) or original.