

Ratio, proportion and rates of change

AQA · GCSE · Mathematics

AQA GCSE Mathematics 8300 topic 3.3 (specification Version 1.0, 12 September 2014, first exams 2017). Foundation rows R1–R16 run in three tiers; the Higher-only additions are compound units (density, pressure) in numerical AND algebraic contexts (R1), working with percentages above 100% and original-value problems (R9), $k : k^2 : k^3$ similarity ratios (R12), constructing equations for direct and inverse proportion (R13), instantaneous rate of change as the gradient of a tangent (R15), and general iterative processes beyond compound interest (R16). The June 2024 Higher set supplies verified anchors below.

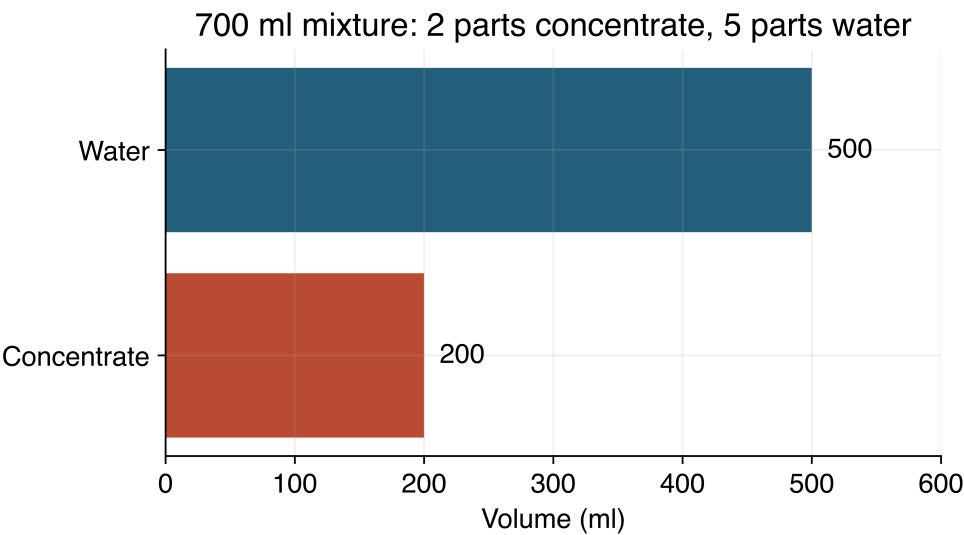
Must-know for topic 3.3:

Family	Formulae
multipliers	increase $r\%$: $\times (1 + \frac{r}{100})$; decrease: $\times (1 - \frac{r}{100})$; original value = final $\div$ multiplier
compound units	speed = $\frac{d}{t}$; density = $\frac{m}{V}$; pressure = $\frac{F}{A}$
proportion	direct $y = kx^n$; inverse $y = \frac{k}{x^n}$ – find k from one pair
growth/decay	$A = P (1 \pm \frac{r}{100})^n$; iterative $x_{n+1} = f(x_n)$

Units, ratio notation and sharing

Change freely between related standard units (time, length, area, volume/capacity, mass) and compound units (speed, rates of pay, prices) in numerical contexts; additional foundation adds **density** and **pressure**, and Higher works them in numerical AND algebraic contexts – a cylinder of volume $V = 2x^3$ and mass $m = 5x^2$ has density $\frac{5x^2}{2x^3} = \frac{5}{2x}$.

Ratio notation reduces to simplest form ($45 : 60 = 3 : 4$); express one quantity as a fraction of another (fraction above or below 1); divide a quantity in a part : part or **part : whole ratio** – £350 in 2 : 5 gives £100 and £250. Express a multiplicative relationship as ratio or fraction (R6); proportion as equality of ratios (R7); relate ratios to fractions and LINEAR functions (R8: the unitary method and $y = kx$ are the same ladder). Apply ratio to real contexts – conversion, comparison, scaling, mixing, concentrations, better value/**best-buy** problems (R5): compare unit prices before deciding.

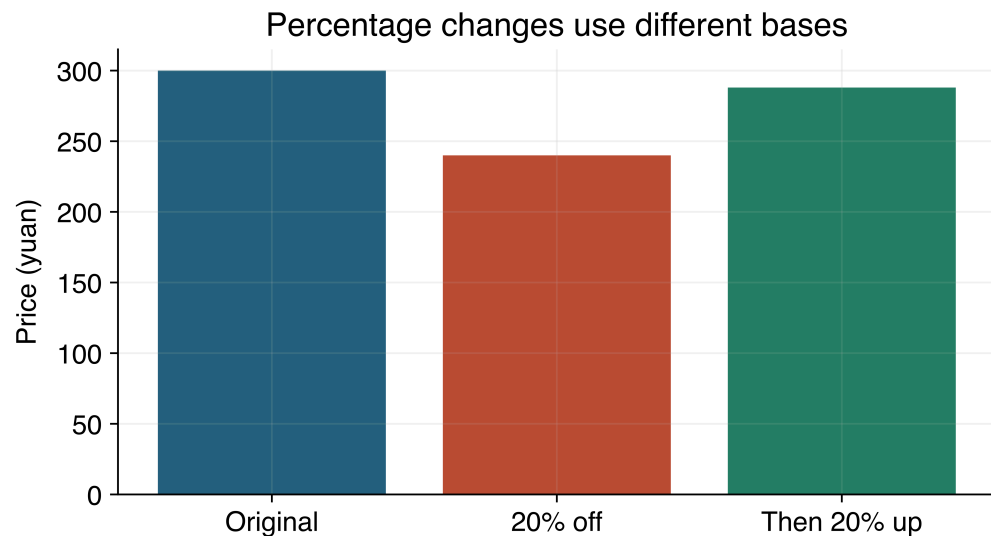


Original native-lesson illustration; labels belong to its worked example.

Percentages and percentage change

Percentage is parts per hundred; interpret percentages and percentage changes as fractions/decimals and MULTIPLICATIVELY (R9). Additional foundation: express one quantity as a percentage of another, compare two quantities, work above 100%, percentage increase/decrease, original-value problems and simple interest. The **multiplier** discipline: +12% is $\times 1.12$; -17% is $\times 0.83$; original value divides the final by the multiplier.

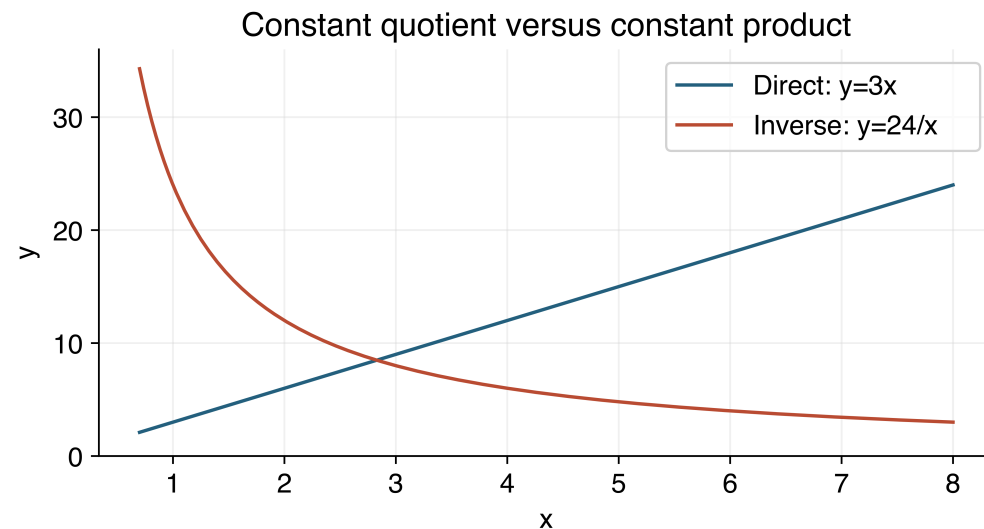
Worked example (2H June 2024 Q2, adapted). A town’s population fell by 12% to 2200. Original = $2200 \div 0.88 = 2500$. The MS’s own multiplier check: $2,200,000 \div 0.88 = 2.5 \times 10^6$ in standard form – same division, scaled-up numbers.



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Direct and inverse proportion

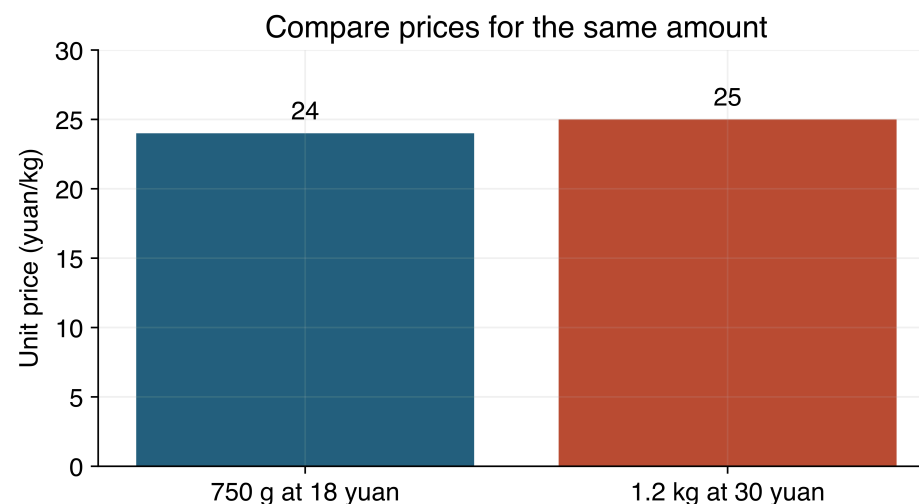
Solve direct and inverse proportion problems with graphical and ALGEBRAIC representations (R10). Higher interprets and CONSTRUCTS equations describing proportion (R13): $X \propto Y$ means $X = kY$; X inversely proportional to Y means $X = k/Y$. If $y \propto x^3$ with $y = 24$ at $x = 2$, then the **constant of proportionality** is $k = 3$; if $p \propto \frac{1}{x^2}$ with $p = 8$ at $x = 2$, then $k = 32$ and $p = \frac{32}{x^2}$. Recognise and interpret the graphs: **direct proportion** is a straight line through the origin (higher powers curve upward); **inverse proportion** is a reciprocal-shaped curve (R14 links the gradient of a straight line to a rate of change).



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Rates and compound units

Use compound units – speed, rates of pay, unit pricing (Foundation additional); density and pressure (R11), including comparisons. A 234 km journey in 2 h 36 min averages $234 \div 2.6 = 90$ km/h; £9.20/h for 7.5 h pays £69; best-buy comparisons divide price by quantity and compare like with like.

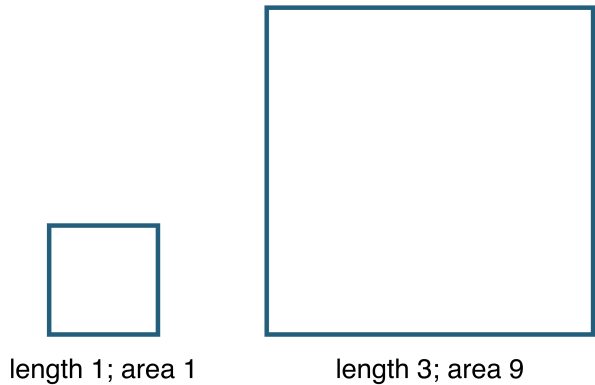


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Scale factors, similarity and gradients

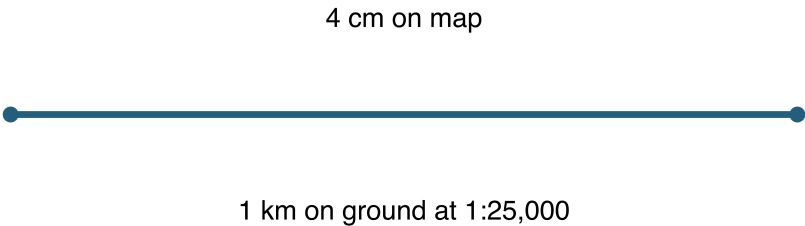
Use **scale** factors, scale diagrams and maps (R2, incl. geometrical problems). Compare lengths, areas and volumes using ratio notation and scale factors (R12): lengths k : **area scale factors** k^2 : volumes k^3 , with links to similarity including trigonometric ratios; Higher extends to the $k : k^2 : k^3$ ladder in problems – similar statues of heights 3 : 5 have volumes in ratio 27 : 125, so a 216 cm³ smaller statue scales to $216 \times \frac{125}{27} = 1000$ cm³. Interpret the gradient of a straight-line graph as a **rate of change** (R14) – on a distance-time graph it is speed; on a velocity-time graph the gradient is acceleration and the AREA is distance. Higher (R15) reads the gradient at a point on a CURVE as the instantaneous rate of change: draw the tangent and take its **gradient** as rise/run – and applies average (chord) and instantaneous (tangent) rates in numerical, algebraic and graphical contexts.

A length factor of 3 acts in both area dimensions



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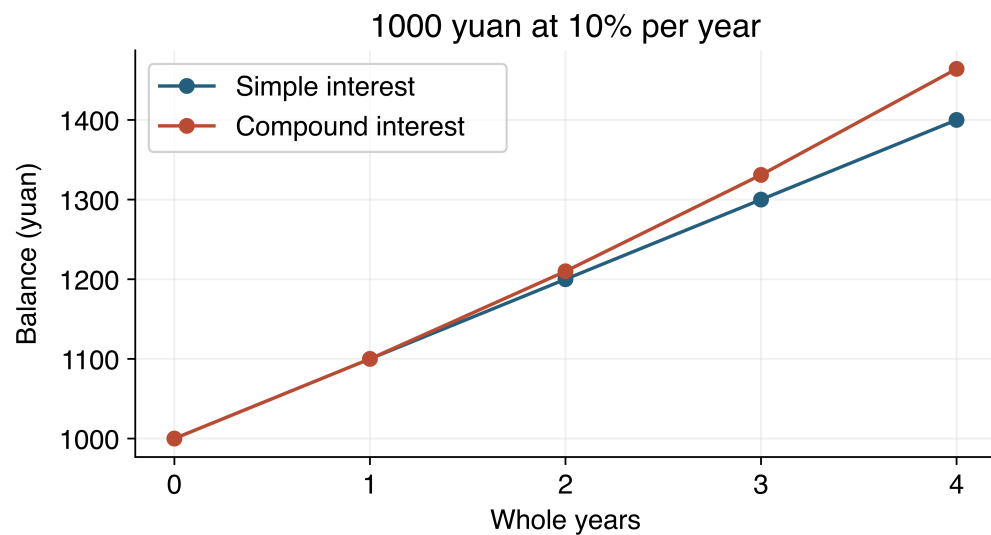
Match units before applying the scale



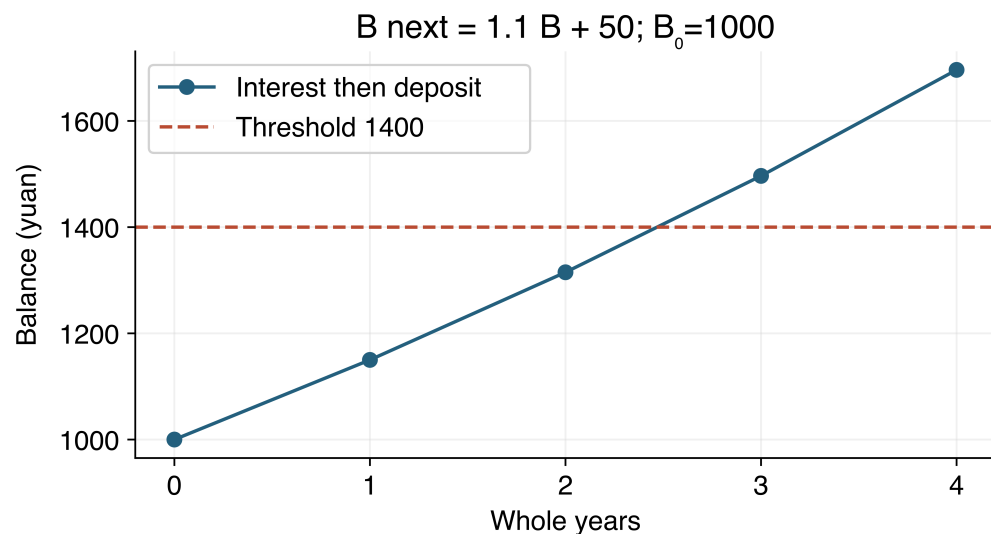
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Growth, decay and iterative processes

Set up, solve and INTERPRET growth and decay problems including **compound interest** (R16): $A = P \left(1 \pm \frac{r}{100}\right)^n$. £2000 at 3% compound for 4 years grows to $2000 \times 1.03^4 = £2251.02$; a car worth £12000 losing 15%/year falls to $12000 \times 0.85^3 = £7369.50$. **Worked example (2H June 2024 Q10, adapted).** A house worth £90000 rises 3% then 8%: $90000 \times 1.03 \times 1.08 = £100116$ – above £100000, so yes. The MS’s multiplier route: $1.03 \times 1.08 = 1.1124 > 1.111\dots = 100000/90000$. Higher adds general **iterative processes** : $x_{n+1} = f(x_n)$ cycles for repeated percentages – a quantity gaining 5% each cycle iterates $x_{n+1} = 1.05x_n$, and the limit (if any) solves $x = f(x)$. Interpret answers in context: round sensibly and state the unit.



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Checklist

- Unit and compound-unit conversions incl. density and pressure, algebraic at Higher (R1, R11).
- Scale factors, diagrams and maps (R2); quantities as fractions; ratio notation, sharing, real-context and best-buy problems (R3–R5).
- Multiplicative relationships as ratio/fraction; proportion as equality of ratios; ratios to fractions and linear functions (R6–R8).
- Percentage multipliers incl. above 100%, change, original value, simple interest (R9; Q2 anchor).
- Direct and inverse proportion graphically and algebraically; interpret and construct proportion equations (R10, R13).
- Gradients as rates of change; chord (average) and tangent (instantaneous) rates (R14–R15).
- Length : area : volume ratios $k : k^2 : k^3$ with similarity links (R12).
- Growth and decay incl. compound interest; general iterative processes with interpretation (R16; Q10 anchor).

Boundary notes: the Formulae Sheet gives nothing for this topic; best-buy comparisons must show unit prices; Higher iterative processes include convergence checks. Worked examples are adapted from the June 2024 Higher papers (see the meta's assessment evidence) or original.