

Algebra

AQA · GCSE · Mathematics

AQA GCSE Mathematics 8300 topic 3.2 (specification Version 1.0, 12 September 2014, first exams 2017). Foundation rows run A1–A25 in three tiers (basic foundation / additional foundation / Higher only); the Higher-only additions are surds and algebraic fractions (A4), identities and proof (A6), inverse and composite functions with f , fg and f^{-1} notation (A7), perpendicular lines (A9), turning points by completing the square (A11), cubic, reciprocal, exponential and trigonometric graphs (A12), the circle equation with tangents (A16), the quadratic formula (A18), linear/quadratic simultaneity (A19), iteration (A20), set-notation and graphical inequality regions (A22) and quadratic/geometric/surd sequences (A24–A25). The June 2024 Higher set supplies verified anchors below.

Must-know for topic 3.2 (the Formulae Sheet gives only the quadratic formula):

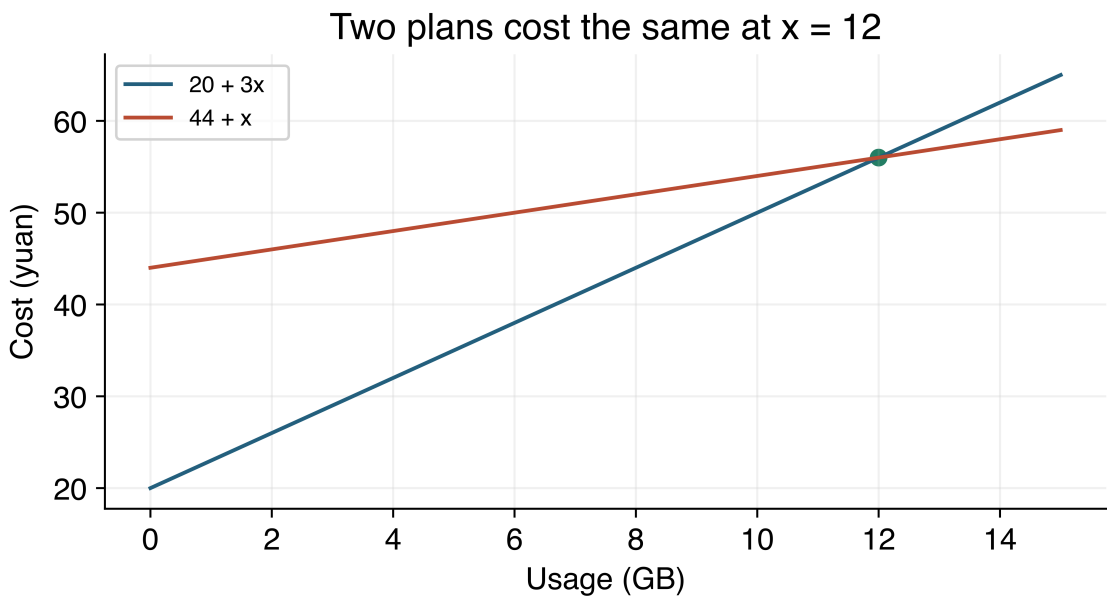
Family	Formulae
quadratic	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$; $a(x + b)^2 + c$ gives the turning point
lines	$m = \frac{y_2 - y_1}{x_2 - x_1}$; perpendicular $m_1 m_2 = -1$; $y - y_1 = m(x - x_1)$
iteration	$x_{n+1} = f(x_n)$ from a rearranged equation, with suffix notation
nth terms	linear $an + b$; quadratic $an^2 + bn + c$ with $a = \frac{1}{2}(\text{second difference})$

Notation, vocabulary and manipulation

Algebraic notation writes ab for $a \times b$, $3y$ for $y + y + y$, a^2 , a^3 , a^2b , $\frac{a}{b}$ for $a \div b$ – coefficients as fractions, not decimals – and brackets; answers are expected in simplest form without being told. Substitute into formulae including scientific ones (unseen formulae are quoted in the question). The vocabulary – expression, equation, formula, inequality, identity, term, factor – carries meaning: an equation asks which values work; an identity holds for every value.

Simplify and manipulate by collecting like terms, multiplying a single term over a bracket, taking out common factors and using the index laws. Foundation additional adds expanding two binomials and factorising $x^2 + bx + c$ with the difference of two squares. Higher adds surds and **algebraic fractions**: expand and simplify $(3 + \sqrt{2})(1 - 2\sqrt{2}) = 3 - 6\sqrt{2} + \sqrt{2} - 4 = -1 - 5\sqrt{2}$; factorise $6x^2 - 7x - 3 = (3x + 1)(2x - 3)$.

An **identity** argues equivalence for ALL values: $(x + 2)^2 \equiv x^2 + 4x + 4$, and the 3H June 2024 anchor asks exactly this ("it is true for all values of x "). Proofs expand and compare both sides – numeric examples never prove an identity. Rearrange formulae to change the subject ($v = u + at \Rightarrow t = \frac{v-u}{a}$), including from other subjects in words or symbols.



Original native-lesson illustration; labels belong to its worked example.

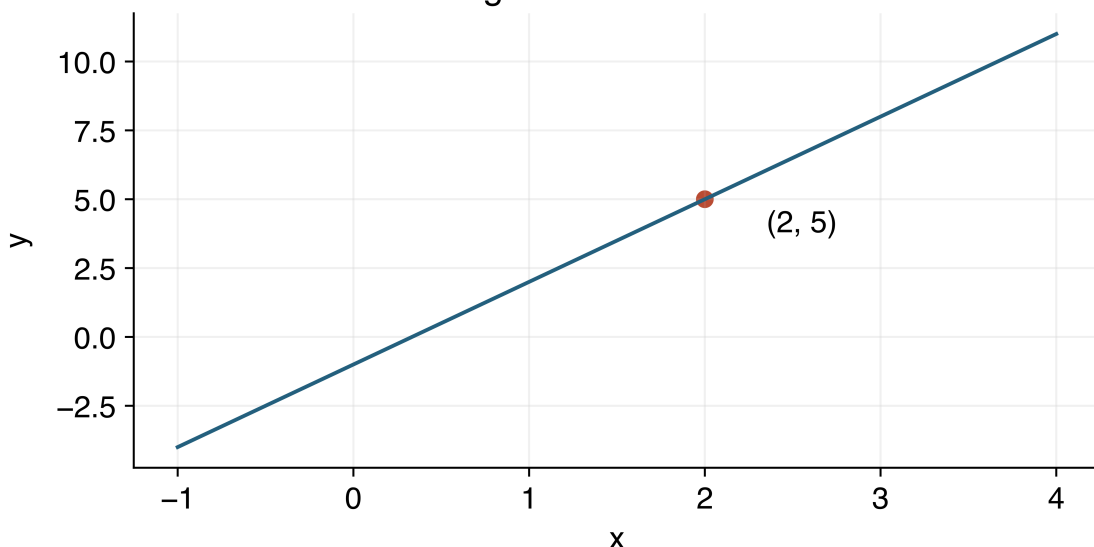
Functions

A function maps inputs to outputs; interpret simple expressions as functions (A7, Foundation). Higher adds the **inverse function** – reverse the operations, or swap and rearrange – and the **composite** $fg(x) = f(g(x))$, apply g first, using $f(x)$, $fg(x)$ and $f^{-1}(x)$ notation. Worked case: $f(x) = 3x - 5$, $g(x) = x^2$: $fg(2) = f(4) = 7$; from $y = 3x - 5$, $x = \frac{y+5}{3}$, so $f^{-1}(x) = \frac{x+5}{3}$. Check $f^{-1}(f(x)) = x$.

Graphs: lines to curves

Coordinates in all four quadrants; plot straight lines from $y = mx + c$. The additional foundation tier identifies parallel lines from equal gradients; Higher finds the equation through two points or one point with a given gradient and identifies perpendicular lines ($m_1m_2 = -1$): through (1,7) and (2,9) the line is $y = 2x + 5$, and the perpendicular through (1,7) is $y - 7 = -\frac{1}{2}(x - 1)$. Identify **gradients** and intercepts graphically AND algebraically (A10).

Point and gradient determine the line



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For quadratics (A11) identify roots, intercepts and turning points graphically; additional foundation deduces roots algebraically; Higher deduces turning points by completing the square : $y = x^2 - 6x + 5 = (x - 3)^2 - 4$ has minimum $(3, -4)$ and the symmetry $x = 3$. Recognise and sketch linear and quadratic graphs (A12); the additional tier adds simple cubics and the **reciprocal** $y = \frac{1}{x}$ ($x \neq 0$); Higher adds exponentials $y = k^x$ ($k > 0$) and $y = \sin x$, $\cos x$, $\tan x$ for angles of any size. Sketch translations and reflections of a given function (A13) – $y = f(x) + a$ up, $y = f(x + a)$ left. Plot and interpret non-standard real-context graphs including kinematic distance/speed/acceleration (A14, reciprocal and exponential at Higher); estimate gradients and areas under graphs and interpret them as rates and distances (A15 – a velocity-time area is distance).

Higher A16: the circle centred at the origin has $x^2 + y^2 = r^2$, and the tangent at (x_1, y_1) has gradient $-\frac{x_1}{y_1}$ (radius gradient $\frac{y_1}{x_1}$; product -1).

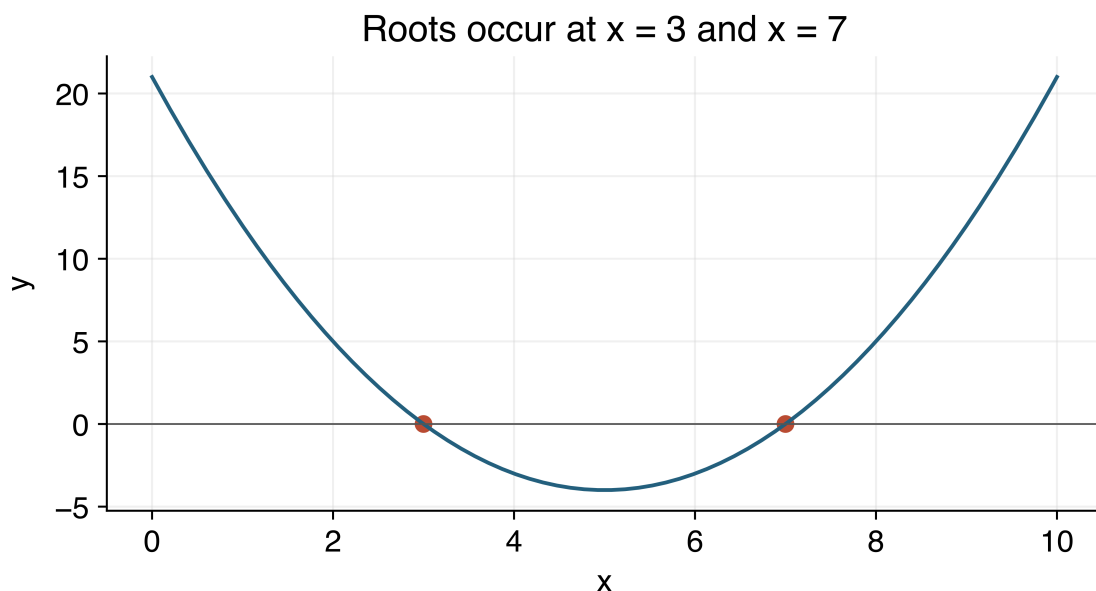
Solving equations and inequalities

Solve linear equations in one unknown (A17) – unknown on either side, brackets included: $7x - 22 = 4x + 29$ gives $3x = 51$ and $x = 17$ (the 1H June 2024 anchor Q8). Quadratics (A18): factorise $x^2 + bx + c$ (Foundation additional, including rearrangement), so each **root** comes from a zero factor; Higher adds completing the square and the quadratic formula, with the **discriminant** deciding the root count – $x^2 + 6x + 2 = 0$ gives $x = -3 \pm \sqrt{7}$. Approximate solutions from graphs at every tier. Simultaneity (A19): two linear equations algebraically and graphically by **elimination** (Foundation); Higher adds linear/quadratic – $y = 2x + 5$ with

$y = x^2$: substituting gives $x^2 - 2x - 5 = 0$, so $x = 1 \pm \sqrt{6}$ – keep both intersections.

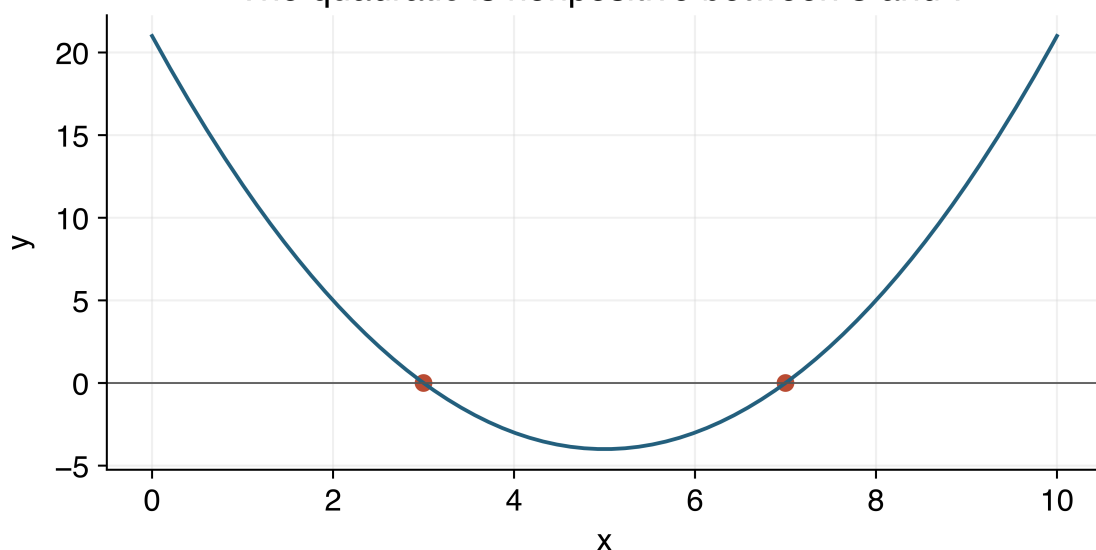
Iteration (A20, Higher): solve numerically with suffix notation – from $x^3 - 3x - 1 = 0$, rearrange $x = \sqrt[3]{3x + 1}$ and iterate $x_{n+1} = \sqrt[3]{3x_n + 1}$ from $x_1 = 1$: $x_2 = 1.587$, $x_3 = 1.793$, $x_4 = 1.855$, $x_5 = 1.872$, converging to the root 1.879 (3 d.p.). Show values to a stated accuracy and check the final value back in the original equation.

Translate situations into algebra (A21) – derive and solve, including geometry and context problems. Inequalities (A22): solve linear inequalities in one variable (Foundation) with number-line representation (open circle strict, closed circle included); Higher solves linear inequalities in one or two variables and quadratic inequalities, with **set notation** and graphical regions – $x^2 - x - 6 > 0$ factors to $(x - 3)(x + 2) > 0$, so $x < -2$ or $x > 3$, written $\{x: x < -2\} \cup \{x: x > 3\}$; dashed boundary lines for strict inequalities.



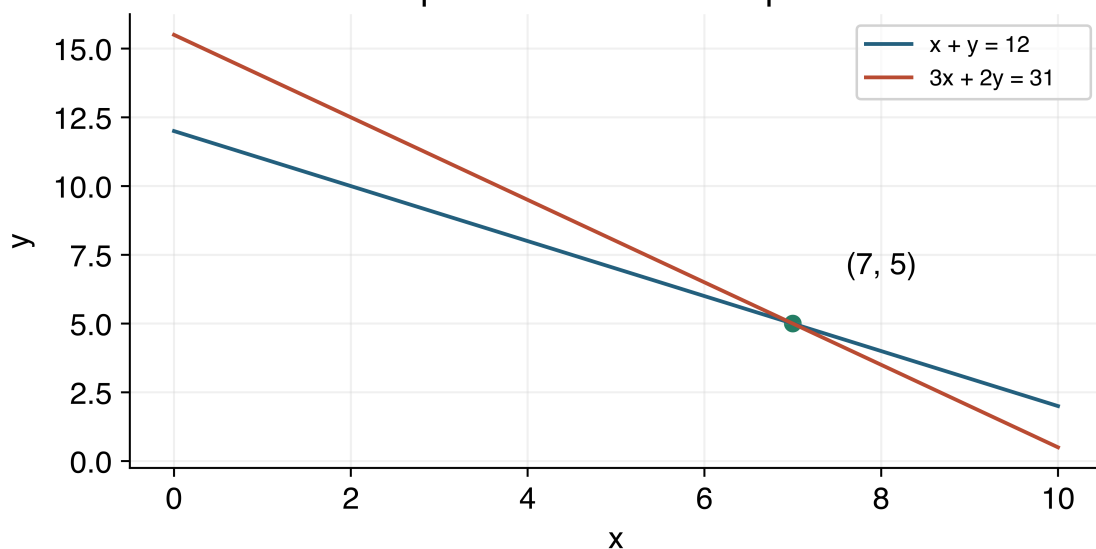
Original native-lesson illustration; labels belong to its worked example.

The quadratic is nonpositive between 3 and 7



Original native-lesson illustration; labels belong to its worked example.

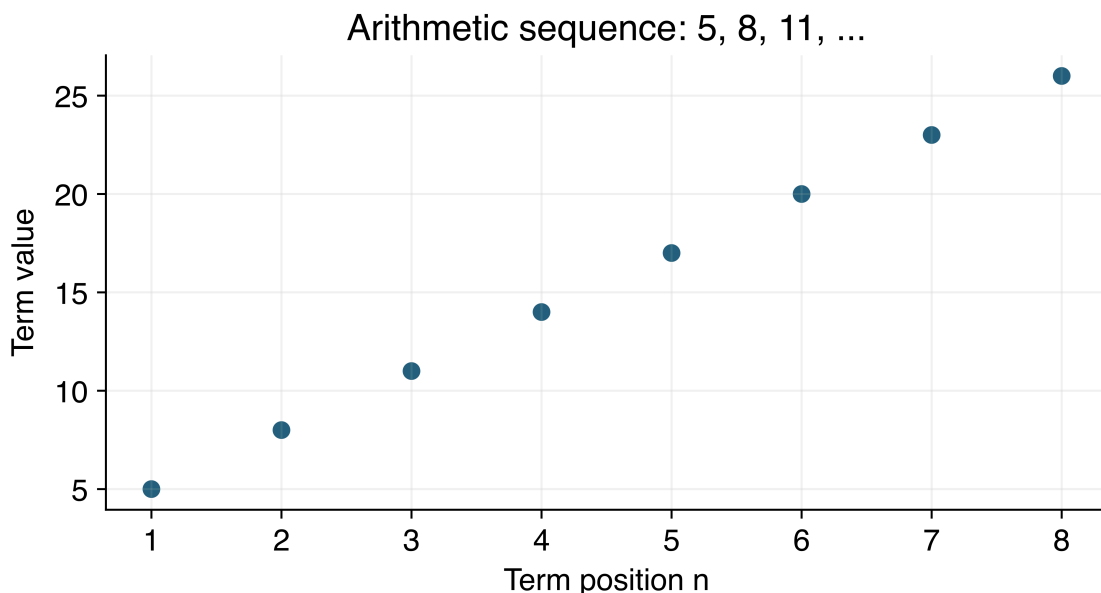
One point satisfies both equations



Original native-lesson illustration; labels belong to its worked example.

Sequences

Generate terms from term-to-term and position-to-term rules, including patterns and diagrams (A23). Recognise triangular, square and cube numbers and simple arithmetic **progressions** (A24); additional foundation adds Fibonacci-type sequences, quadratic sequences and geometric **progressions** ar^n (r a positive rational); Higher adds other/recursively-defined sequences and surd ratios. Deduce **nth terms** (A25): linear $an + b$ (Foundation), quadratic $an^2 + bn + c$ at Higher – for 5, 9, 17, 29, 45, ... the second difference is 4, so $a = 2$ and the sequence is $2n^2 - 2n + 5$ (check $n = 4$: $32 - 8 + 5 = 29$).



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Checklist

- Notation ab , $3y$, a^2 , $\frac{a}{b}$; substitution into scientific formulae; vocabulary including identity (A1–A3).
- Collect terms, brackets, common factors, index laws; two binomials and $x^2 + bx + c$ (Foundation); surds and algebraic fractions (Higher) (A4).
- Standard formulae and changing the subject (A5); identities, arguments and proofs (A6).
- Functions with inputs/outputs; inverse and composite with f , fg , f^{-1} notation (A7, Higher).
- Coordinates, plotting, parallel lines (Foundation); equation through two points/point+gradient, perpendicular lines (Higher) (A8–A9).

- Gradients and intercepts both ways (A10); roots, intercepts, turning points – by completing the square at Higher (A11).
- Sketch linear, quadratic, cubic, reciprocal, exponential and trig graphs; translations/reflections; real-context and kinematic graphs; gradient/area interpretation (A12–A15).
- Circle $x^2 + y^2 = r^2$ with tangent gradients (A16, Higher).
- Linear equations both sides (Q8 anchor); quadratics by factorising, formula, completing the square; linear and linear/quadratic simultaneity keeping both roots (A17–A19).
- Iteration with suffix notation to a stated accuracy (A20); modelling and context equations (A21); inequalities with number lines, set notation and graphical regions (A22).
- Sequence types incl. Fibonacci, quadratic, geometric with surd ratios; linear and quadratic nth terms (A23–A25).

Boundary notes: the Formulae Sheet provides the quadratic formula; iteration answers must state the accuracy; graphical inequality conventions (dashed strict, solid included) are required. Worked examples above are adapted from the June 2024 Higher papers (see the meta’s assessment evidence) or original.