Number – native teaching supplement

AQA · GCSE · Mathematics

Supported teaching and tier boundary

8300: Number. Version: Version 1.0, 12 September 2014; first examination 2017.

Foundation teaching and Higher additions are labelled below. This reference packages the existing native-lesson crosswalk. It does not certify unreviewed specification rows or a whole qualification. Original diagnostics are separate and are not reproduced.

■ Exact arithmetic and estimation · Foundation

Prime factors reveal shared structure. Use the smallest common prime powers for the HCF and the largest for the LCM. Estimate before calculating; use brackets to preserve the order of operations.

$$72 = 2^3 \times 3^2, \quad 90 = 2 \times 3^2 \times 5, \quad \text{HCF} = 18$$

72=2^3×3^2 and 90=2×3^2×5. Their HCF is 2×9=18. Make 18 bags with 4 pencils and 5 pens each. Their LCM is 2^3×3^2×5=360. A prime integer is greater than 1 and has exactly two positive factors. The number 1 is not prime; a composite positive integer greater than 1 has more than two positive factors. Every integer greater than 1 has a unique product of prime factors apart from their order. For example, $60=2^2\times3\times5$. A factor divides a number; a multiple is the result of multiplying it by an integer. HCF uses common smallest powers, while LCM uses largest powers.

The HCF divides both numbers; the LCM is a multiple of both. They answer different questions. A decimal estimate is not an exact fraction.

For a non-calculator paper, keep fractions exact and show cancellation. For a calculator paper, enter the full expression and compare with your estimate.

■ Exact arithmetic and estimation · Higher

Prime factors reveal shared structure. Use the smallest common prime powers for the HCF and the largest for the LCM. Estimate before calculating; use brackets to preserve the order of operations.

$$a = \prod p_i^{\alpha_i}, \quad b = \prod p_i^{\beta_i}, \quad \text{HCF}(a,b) = \prod p_i^{\min(\alpha_i,\beta_i)}$$

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$72 = 2^3 \times 3^2$

$90 = 2 \times 3^2 \times 5$

Shared smallest powers: $2 \times 3^2 = 18$

Original native-lesson illustration; labels belong to its worked example.

■ Integer indices and standard form · Foundation

Use integer powers, square and cube roots and standard form with 1≤a<10. In multiplying powers with the same base, add indices; in division, subtract them.

$$a^m a^n = a^{m+n}$$

0.000072=7.2×10^(-5). Also $2^3\times2^4=2^7=128$. The square root of 81 is 9. Check a standard-form answer by writing it out as a decimal. Useful powers include $3^3=27$, $4^3=64$, $5^3=125$, $15^2=225$ and $10^6=1{,}000{,}000$. Integer laws give $2^0=1$, 2^(-3)=1/8 and $(2^3)^2=2^6=64$. For standard-form multiplication, (3×10^5)(4×10^(-3))$=12\times10^2=1.2\times10^3$. Division gives $(6\times10^5)/(2\times10^2)=3\times10^3$. For addition, first align exponents: $3\times10^4+2\times10^3=3.2\times10^4$.

Index laws do not turn a sum into a single power: 2^3+2^4=24, not 2^7. Do not round a surd when an exact answer is requested.

This Foundation/Core lesson excludes fractional powers and surd rationalisation. Estimate a result before using a calculator and retain the required precision.

■ Indices, surds and standard form · Higher

For the same positive base, multiplication adds indices and division subtracts them. A negative index means reciprocal; a fractional index represents a root. Standard form has $1 \le a < 10$.

$$a^m a^n = a^{m+n}, \quad a^{-n} = \frac{1}{a^n}, \quad a^{m/n} = \left(\sqrt[n]{a}\right)^m$$

$0.000072 = 7.2 \times 10^{-5}$. Also $16^{3/4} = (16^{1/4})^3 = 2^3 = 8$. Simplify $\sqrt{72} = 6\sqrt{2}$, then rationalise $1/\sqrt{2} = \sqrt{2}/2$. Useful powers include $3^3=27$, $4^3=64$, $5^3=125$, $15^2=225$ and $10^6=1{,}000{,}000$. Integer laws give $2^0=1$, $2^{-3}=1/8$ and $(2^3)^2=2^6=64$. For standard-form multiplication, $(3\times10^5)(4\times10^{-3})=12\times10^2=1.2\times10^3$. Division gives $(6\times10^5)/(2\times10^2)=3\times10^3$. For addition, first align exponents: $3\times10^4+2\times10^3=3.2\times10^4$. For a positive base, $a^{m/n}$=(the nth root of a)^m. Thus $27^{2/3}=3^2=9$ and $16^{-1/2}=1/4$. To estimate $\sqrt{20}$ without a calculator, $4^2<20<5^2$ gives $4<\sqrt{20}<5$; testing $4.5^2=20.25$ shows $\sqrt{20}$ is just below 4.5. Do not use a rough estimate as an exact surd answer.

Index laws do not turn a sum into a single power: $2^3+2^4=24$, not 2^7. Do not round a surd when an exact answer is requested.

Check powers of ten against the original quantity. Use surds for exact geometry, and round only the final length when the question asks for a decimal.

$0.000072 = 7.2 \times 10^{-5}$

7.2 is the coefficient; -5 is the power

The coefficient is between 1 and 10

Original native-lesson illustration; labels belong to its worked example.

■ Accuracy, bounds and compound measures · Higher

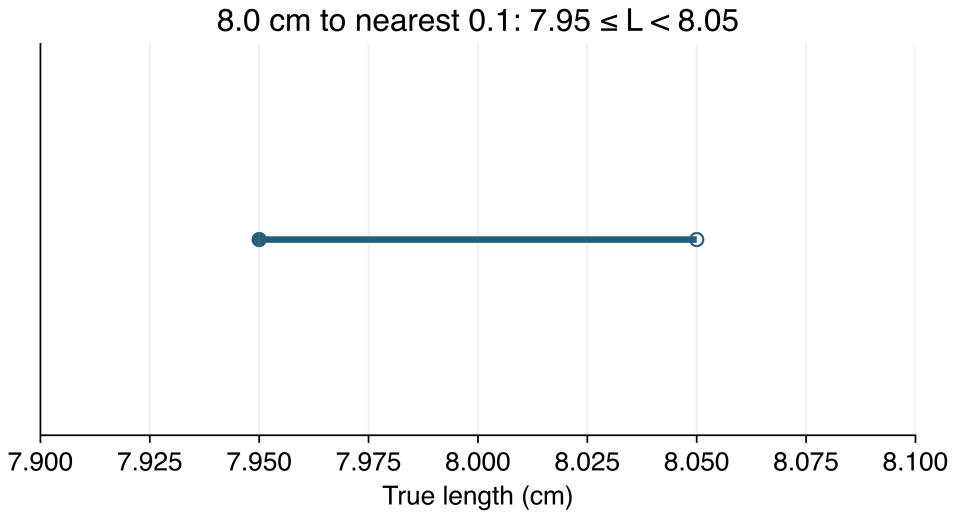
A value rounded to the nearest unit u lies from stated value-u/2 up to, but usually not including, stated value+u/2. For positive quantities, combine extremes according to the operation.

$$A = LW, \qquad v = \frac{d}{t}$$

The lengths satisfy $7.95 \le L < 8.05$ and $4.95 \le W < 5.05$. Since A=LW, $39.3525 \le A < 40.6525$. For speed d/t, the largest speed uses the largest distance and smallest positive time.

An upper bound is not automatically achieved. Dividing upper distance by upper time does not give the largest speed. Keep enough digits in intermediate calculations.

Distinguish measurement uncertainty from arithmetic rounding. A sensible reported precision cannot be finer than the measurements justify.



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■ Signed numbers, place value and operation order · Foundation

Larger numbers lie farther right on the number line. Adding a negative moves left; subtracting a negative moves right. Multiply or divide signs first, then magnitudes. Evaluate brackets, powers and roots before multiplication/division, then addition/subtraction, working left to right within equal priority.

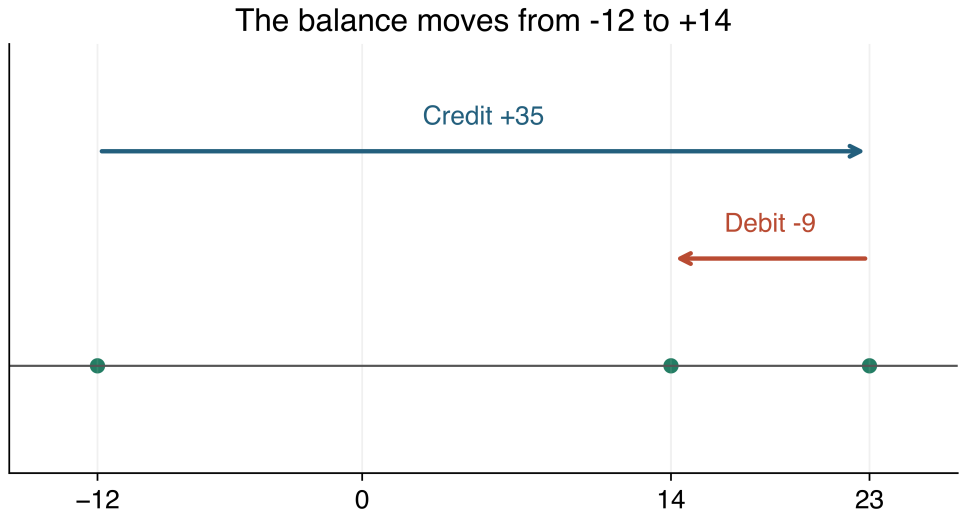
$$-12 + 35 - 9 = 14, \quad 18 \div 3 \times 2 = 12$$

The balance is -12+35-9=14 yuan. Also -4-(-7)=3 and $(-6)\times(-3)=18$. The reciprocal

of -4 is -1/4, since their product is 1. For 18÷3×2, work left to right to obtain 12. In 3.047, the 4 means four hundredths and the 7 means seven thousandths. For decimal multiplication, first calculate 24×35: 24×30+24×5=720+120=840. The original 2.4×0.35 has three decimal places in total, giving 0.840. For 5.04÷0.12 multiply both numbers by 100 to get 504÷12; 12×40=480 leaves 24, so the answer is 42. To order negative fractions, -3/4=-0.75 and -4/5=-0.8, hence -4/5<-3/4. The symbols ≤ and ≥ include equality; ≠ means unequal.

Subtraction is not commutative: 3-8 and 8-3 differ. A negative sign outside a square is not inside its base: $-3^2=-9$, but $(-3)^2=9$. Zero has no reciprocal.

Use a signed starting balance, a positive credit and a negative debit. A shop buying for 48 yuan and selling for 60 makes 12 yuan profit; reversing the prices gives a 12-yuan loss. Estimate the sign before calculating.



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■ Exact fractions and mixed-number operations · Foundation

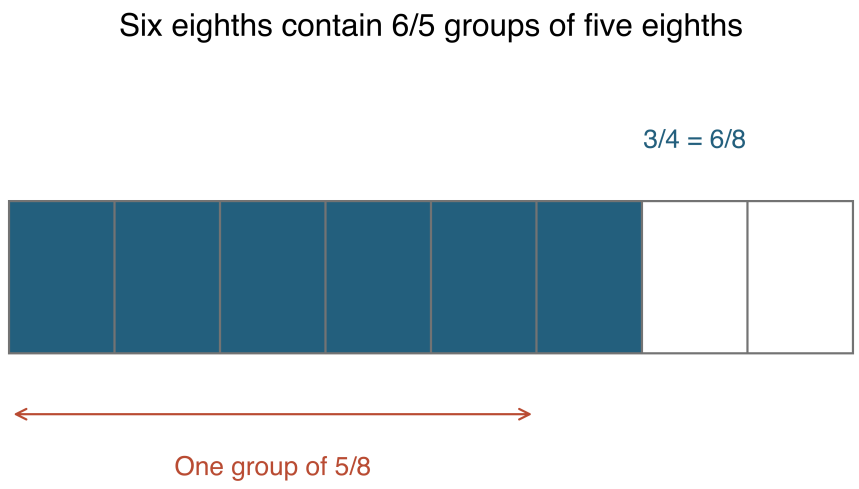
Convert mixed numbers to improper fractions. Add or subtract using a common denominator; multiply numerators and denominators; divide by a nonzero fraction by multiplying its reciprocal. Cancel common factors, not added terms. These rules also apply to negative fractions.

$$\frac{3}{4} \div \frac{5}{8} = \frac{3}{4} \times \frac{8}{5} = \frac{6}{5}$$

1½-¾=6/4-3/4=3/4. Also (-2/3)×(9/4)=-18/12=-3/2 and (3/4)÷(5/8)=(3/4)×(8/5)=6/5. A common denominator gives 5/6+3/4=10/12+9/12=19/12. Check division by multiplying 6/5 by 5/8 to recover 3/4. Exact multiples of π follow ordinary arithmetic: 3π+2π=5π and 6π/3=2π. Leave an answer such as 5π exact when requested; π≈3.14 would introduce approximation. For subtraction, 5/6-3/4=10/12-9/12=1/12; for a negative mixed number, -1½ means -(1+1/2)=-3/2.

Adding denominators does not preserve the unit size: 1/2+1/3 is not 2/5. Cancel only factors of a whole numerator and denominator. A division by zero is undefined.

For a non-calculator question show the common denominator or reciprocal step. Convert 19/12 to 1 7/12 if a mixed number is requested; round only when the question explicitly needs a decimal.



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■ Systematic lists and possibility grids · Foundation

Fix one first choice and list every allowed second choice before moving to the next first choice. Use a table to check that no outcome is missing or duplicated. If combinations are forbidden, remove those entries from the list.

$$\{TA, TB, TM, JA, JB, JM\}$$

List tea-apple, tea-banana, tea-melon, juice-apple, juice-banana, juice-melon: six choices. Removing tea-melon leaves five. The codes using 1,2,3 once each are 123,132,213,231,312,321: six. These answers follow from complete lists.

State whether order matters and whether repetition is allowed. Choosing A then B can be different from B then A for a code, but not for an unordered two-person team. A restriction can make a simple product invalid.

Foundation should justify answers with a complete ordered list or grid. Higher may compress a verified structure using the product rule, but must still check restrictions.

■ Systematic lists and the product rule · Higher

Fix one first choice and list every permitted second choice before moving to the next first choice. A table gives the same structure. When every one of m first choices allows n second choices, the Higher product rule gives mn outcomes. If restrictions change the options, count the allowed rows separately.

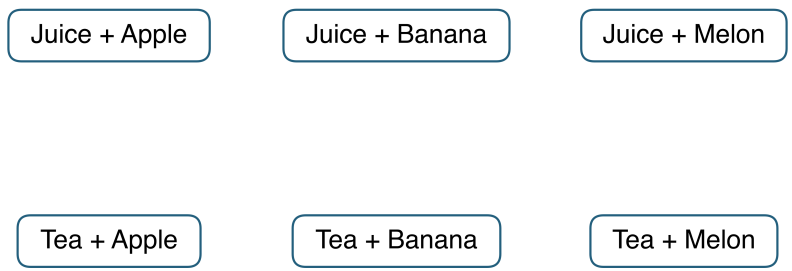
$$\{TA, TB, TM, JA, JB, JM\}$$

The list is tea-apple, tea-banana, tea-melon, juice-apple, juice-banana, juice-melon: six outcomes. If tea-melon is unavailable, five remain. For a three-digit code with digits 1,2,3 and no repeats, choose 3 then 2 then 1 possibilities, giving six codes: 123,132,213,231,312,321.

State whether order matters and whether repetition is allowed. Choosing A then B can be different from B then A for a code, but not for an unordered two-person team. A restriction can make a simple product invalid.

Foundation should justify answers with a complete ordered list or grid. Higher may compress a verified structure using the product rule, but must still check restrictions.

A complete grid lists six choices once each



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■ Exact surds and rationalising denominators · Higher

Extract square factors: $\sqrt{a^2b}=a\sqrt{b}$ for $a\geq0, b\geq0$. Add only matching root parts. Multiply roots with nonnegative radicands. To rationalise a denominator, multiply numerator and denominator by the same suitable root or conjugate.

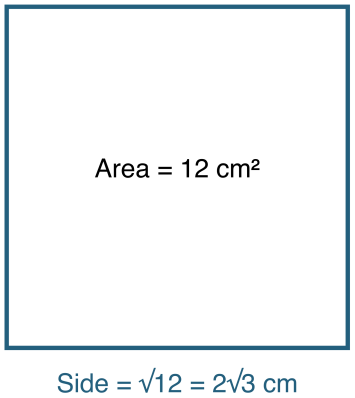
$$\sqrt{12}+\sqrt{27}=5\sqrt{3}, \quad \frac{6}{\sqrt{3}}=2\sqrt{3}$$

The side is $\sqrt{12}=\sqrt{(4\times3)}=2\sqrt{3}$ cm. Thus $\sqrt{12}+\sqrt{27}=2\sqrt{3}+3\sqrt{3}=5\sqrt{3}$. Also $6/\sqrt{3}=6\sqrt{3}/3=2\sqrt{3}$. For $1/(2+\sqrt{3})$, multiply by $(2-\sqrt{3})/(2-\sqrt{3})$: the denominator becomes $4-3=1$, giving $2-\sqrt{3}$. A circle of radius 3 has exact area 9π; a decimal is an approximation.

$\sqrt{(a+b)}$ generally differs from $\sqrt{a}+\sqrt{b}$. Match the radicand before collecting terms. Rationalising changes the form, not the value; multiplying only the denominator changes the value.

AQA N8 Higher requires exact surds and rationalisation. Foundation retains exact fractions and multiples of π; do not assign this surd lesson to Foundation.

An exact root describes the square



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■ Terminating decimals and fractions · Foundation

Use the place value of the last digit to write a power-of-ten denominator, then simplify. Compare numbers using matching decimal places or a common denominator.

$$0.375=\frac{375}{1000}=\frac{3}{8}$$

0.375=375/1000=3/8. Also 3.5=35/10=7/2 and 0.06=6/100=3/50. To order 3/8 and 0.4, write 0.375 and 0.400; therefore 3/8<0.4.

Keep the place value: 0.06 is six hundredths, not six tenths. A fraction may exceed 1. Simplify the entire numerator and denominator by the same common factor.

AQA N10 Foundation covers terminating decimals and fractions, including ordering. Recurring-decimal conversion is reserved for the Higher lesson.

■ Terminating and recurring decimal conversions · Higher

A finite decimal uses a power-of-ten denominator before simplifying. For a recurring decimal, multiply by powers of ten so the repeated tails align, then subtract. Use matching decimal places to compare numbers. A displayed rounded decimal need not be the exact fraction.

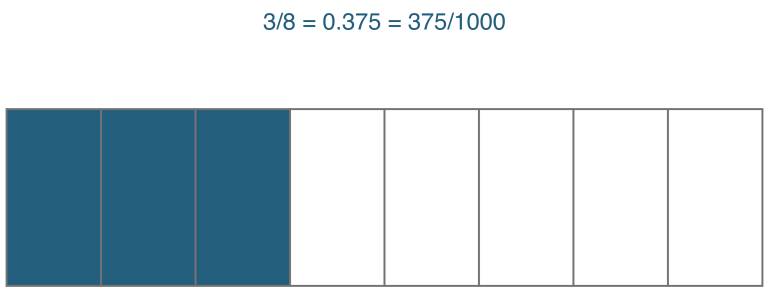
$$100x - x = 27, \quad x = \frac{27}{99} = \frac{3}{11}$$

0.375=375/1000=3/8. For x=0.272727..., 100x=27.272727..., so 99x=27 and x=27/99=3/11. For y=0.16666..., 100y-10y=16.666...-1.666...=15, so y=15/90=1/6. In a reduced fraction, a denominator containing only factors 2 and 5 gives a terminating decimal.

Align the recurring tails before subtracting. 0.333333 is finite and differs from 0.333333... . A non-recurring prefix needs a second power of ten; blindly dividing every digit block by 99 fails.

AQA N10 Higher includes recurring conversion. Both tiers convert and order terminating decimals and fractions. Check a conversion by long division; 3 divided by 8 gives 0.375.

Different exact forms, the same shaded fraction



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■ Metric and compound-unit conversions · Foundation

Convert each dimension. Since 1 m=100 cm, 1 m^2=10,000 cm^2 and 1 m^3=1,000,000 cm^3. Also 1 litre=1000 cm^3, 1 kg=1000 g and 1 hour=3600 seconds. For a compound unit convert numerator and denominator, keeping the physical quantity unchanged.

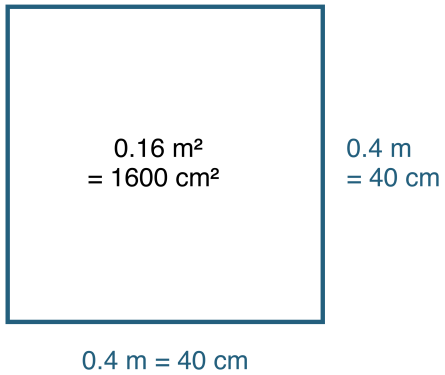
$$1\ \mathrm{m}^2 = 100^2\ \mathrm{cm}^2, \quad 72\frac{\mathrm{km}}{\mathrm{h}} = 20\frac{\mathrm{m}}{\mathrm{s}}$$

0.4 m=40 cm, so the tile area is 40×40=1600 cm^2, agreeing with 0.16×10,000. A 0.002 m^3 container holds 2000 cm^3=2 litres. A speed of 72 km/h is 72,000/3600=20 m/s. Write the units at each stage so the conversion can be checked.

The area multiplier is the square of the length multiplier; the volume multiplier is its cube. An hour is 60 minutes, not 100. Converting only the numerator of a speed gives an inconsistent unit.

AQA N13 and R1 include metric length, area, volume, capacity and compound measures. Use given conversion factors for unfamiliar imperial units; do not guess them.

Convert both dimensions before multiplying



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■ Estimation, rounding and simple error intervals · Foundation

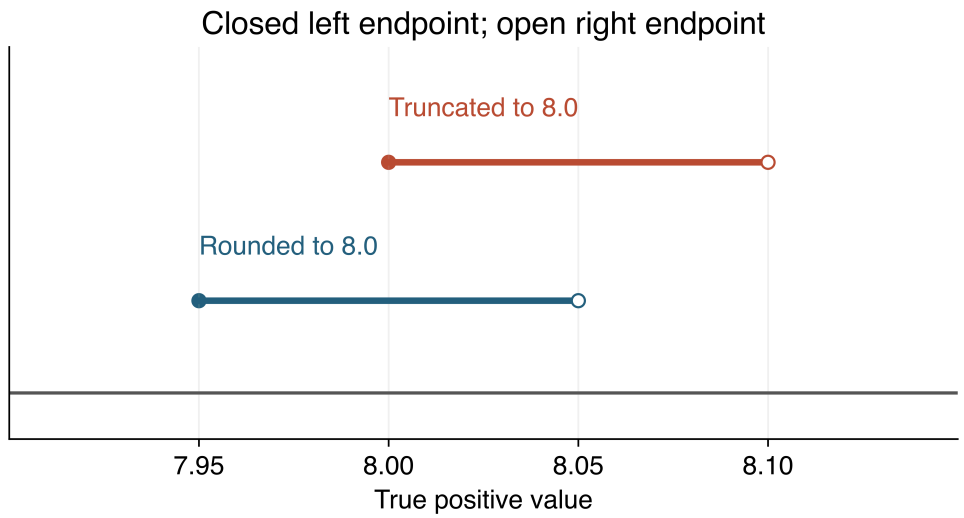
For a rough check use easy nearby numbers: 50×20÷10=100. Decimal places count digits after the point; significant figures start at the first nonzero digit. Round only the final result. Nearest-unit rounding has an interval extending half a unit either way; positive truncation keeps values from the stated value up to the next unit.

$$7.95 \le L < 8.05$$

The calculation is about 96.18, consistent with the estimate 100. The number 0.004786 rounds to 0.0048 at 2 significant figures, but to 0.005 at 3 decimal places. If length L rounds to 8.0 cm at 1 decimal place, 7.95≤L<8.05. If a positive value is truncated to 8.0 at 1 decimal place, 8.0≤L<8.1 instead. Reported precision must fit the question. Seventeen items packed six per box need three whole boxes, since two boxes hold only twelve items. Rounding 17/6 to two boxes would fail the physical requirement. For a nearest-0.1 reading of 8.0, each possible value differs from the report by at most 0.05; a claim of 8.08 lies outside the interval. Carry full calculator precision until a final money, length or accuracy requirement is applied.

Zeros before the first nonzero digit do not count as significant figures. Rounding and truncation give different intervals. Do not turn an approximate check into an exact answer, or round every intermediate result.

AQA N14–N16 require accuracy interpretation at both tiers. Foundation uses simple intervals and limits of accuracy. Higher combines upper and lower bounds for calculated quantities in the separate bounds lesson.



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■ Ratio shares and fraction operators · Foundation

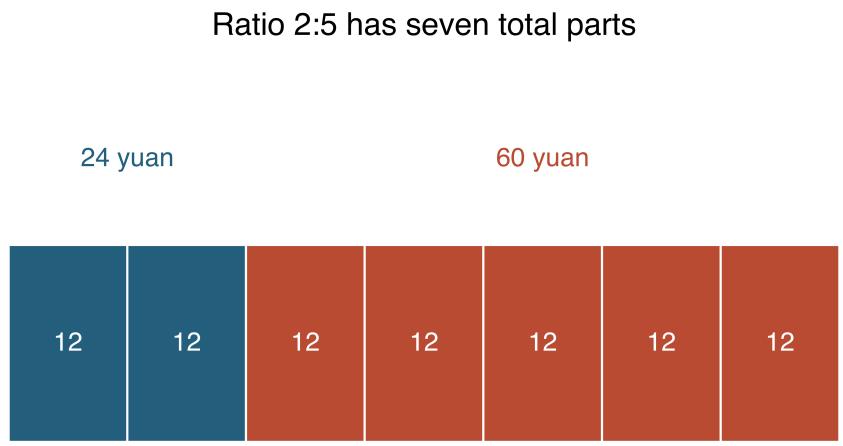
Add the ratio parts to find the whole: 2:5 has 7 equal parts. Divide the total by 7, then multiply by the required part count. A fraction acts as a multiplier; a percentage p acts as p/100. Keep part-to-part and part-to-whole comparisons separate.

$$84 \times \frac{2}{2+5} = 24$$

Each part is 84/7=12 yuan, giving shares 24 and 60. The first team has 2/7 of the whole, while its share is 2/5 of the other share. A 3/4 portion of 28 is 21. A 120% amount of 35 is 1.2×35=42, so percentages may exceed 100.

A ratio 2:5 does not give the first share as 2/5 of the total. The total has 7 parts. A multiplier over 1 increases a positive quantity; it does not automatically mean a probability.

AQA N11/N12 and R3–R8 link ratio parts to fractions and operators. Check the shares add to the stated total and simplify a ratio by dividing every part by the same positive factor.



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Original independent transfer

■ Foundation

A supplier has 84 pencils and 126 pens. Find the greatest number of identical packs using everything. State each pack's contents. A delivery charge of 18.60 CNY is shared equally among six buyers; find each share without rounding intermediate values.

■ Foundation worked solution

$84 = 2^2 \cdot 3 \cdot 7$ and $126 = 2 \cdot 3^2 \cdot 7$, so the HCF is 42. Each of 42 packs has $84/42 = 2$ pencils and $126/42 = 3$ pens. This is maximal because any pack count must divide both totals. Each delivery share is $s = C/n = (18.60 \text{ CNY})/6 = 3.10$ CNY; six shares return the original charge.

Higher

A positive rectangle has recorded length 8.0 cm and width 3.0 cm, each to the nearest 0.1 cm. Give tight lower and upper bounds for area and for length divided by width. State endpoint inclusion.

Higher worked solution

$7.95 \leq l < 8.05$ and $2.95 \leq w < 3.05$, in cm. Positivity makes product and quotient monotonic in the needed directions. Thus $23.4525 \leq A < 24.5525$ in square centimetres. For the dimensionless quotient, $7.95/3.05 < l/w < 8.05/2.95$, or $159/61 < l/w < 161/59$. Neither quotient endpoint is attained, since each requires one excluded upper measurement bound. The area lower bound is attained, since both lower bounds are allowed. Blindly using "upper over lower" without sign and endpoint checks is insufficient.

Terms

prime factor .

index .

lower bound .

reciprocal .

improper fraction .

systematic list .

surd .

terminating decimal .

recurring decimal .

conversion factor .

significant figure .

ratio .