

2.4 Iteration and inequalities

Name: _____ Class: _____ Date: _____

Total: 25 marks

KEY WORDS iteration

Iteration and inequalities

Objective

Find approximate solutions by iteration with suffix notation (A20, Higher); solve linear and quadratic inequalities with number-line, set-notation and graphical representation (A22).

1 Key knowledge and worked examples

Iteration. Rearrange the equation to $x = f(x)$, then cycle $x_{n+1} = f(x_n)$ from a starting value, recording terms to a stated accuracy. From $x^3 - 3x - 1 = 0$: $x_{n+1} = \sqrt[3]{3x_n + 1}$ from $x_1 = 1$ gives 1.587, 1.793, 1.855, 1.872, ... $\rightarrow$ 1.879 (3 d.p.). Check the limit back in the ORIGINAL equation.

Inequalities. Same moves as equations – multiplying/dividing by a negative flips the sign. Number line: open circle strict, closed circle included. Higher: quadratic inequalities and two-variable regions – $x^2 - x - 6 > 0$ gives $x < -2$ or $x > 3$, i.e. $\{x: x < -2\} \cup \{x: x > 3\}$; dashed boundary for strict.

2 Practice

2.1 Solve $3x - 2 < 10$ and represent on a number line [2]

2.2 Solve $7 - x \leq 5$ [2]

2.3 Solve $-7 \leq 3x + 2 < 6$ [3]

2.4 Solve $x^2 - x - 6 > 0$ [3]

2.5 With $x_{n+1} = \frac{1}{2}x_n + 3$ from $x_1 = 1$, find x_2 , x_3 , x_4 and state the limit. [3]

3 Exam-style questions

3.1 (*Original.*) The equation $x^3 - 3x - 1 = 0$ has a root between 1 and 2. (a) Show that rearranging gives $x = \sqrt[3]{3x + 1}$. (b) Use the iteration $x_{n+1} = \sqrt[3]{3x_n + 1}$ from $x_1 = 1$ to find x_4 . (c) Verify the root is 1.879 to 3 d.p. [5]

3.2 (*Original.*) Solve $x^2 + 3x + 2 > 0$ and write the solution set in set notation. [4]

3.3 (*Original.*) Shade or describe the region satisfying $x \geq 0$, $y \geq 1$ and $x + y \leq 5$, stating the vertices. [3]
