1.3.1 Indices

Name: _____ Class: _____ Date: _____

KEY WORDS index

<!– Session 2 native-skill companion –>

Objective

Practise indices within the labelled native lessons. Public original tasks; locally authored answer guidance.

1 Key knowledge and worked examples

■ Integer indices and standard form · Foundation

Use integer powers, square and cube roots and standard form with $1 \le a < 10$. In multiplying powers with the same base, add indices; in division, subtract them.

$$a^m a^n = a^{m+n}$$

$0.000072 = 7.2 \times 10^{-5}$. Also $2^3 \times 2^4 = 2^7 = 128$. The square root of 81 is 9. Check a standard-form answer by writing it out as a decimal. Useful powers include $3^3=27$, $4^3=64$, $5^3=125$, $15^2=225$ and $10^6=1{,}000{,}000$. Integer laws give $2^0=1$, $2^{-3}=1/8$ and $(2^3)^2=2^6=64$. For standard-form multiplication, $(3\times10^5)(4\times10^{-3})=12\times10^2=1.2\times10^3$. Division gives $(6\times10^5)/(2\times10^2)=3\times10^3$. For addition, first align exponents: $3\times10^4+2\times10^3=3.2\times10^4$.

Index laws do not turn a sum into a single power: $2^3+2^4=24$, not 2^7. Do not round a surd when an exact answer is requested.

This Foundation/Core lesson excludes fractional powers and surd rationalisation. Estimate a result before using a calculator and retain the required precision.

■ Indices, surds and standard form · Higher

For the same positive base, multiplication adds indices and division subtracts them. A negative index means reciprocal; a fractional index represents a root. Standard form has $1 \le a < 10$.

$$a^m a^n = a^{m+n}, \quad a^{-n} = \frac{1}{a^n}, \quad a^{m/n} = \left(\sqrt[n]{a}\right)^m$$

$0.000072 = 7.2 \times 10^{-5}$. Also $16^{(3/4)}=(16^{(1/4)})^3=2^3=8$. Simplify $\sqrt{72}=6\sqrt{2}$, then rationalise $1/\sqrt{2}=\sqrt{2}/2$. Useful powers include $3^3=27$, $4^3=64$, $5^3=125$, $15^2=225$ and $10^6=1{,}000{,}000$. Integer laws give $2^0=1$, $2^{-3}=1/8$ and $(2^3)^2=2^6=64$. For standard-form multiplication, $(3\times10^5)(4\times10^{-3})=12\times10^2=1.2\times10^3$. Division gives $(6\times10^5)/(2\times10^2)=3\times10^3$. For addition, first align exponents: $3\times10^4+2\times10^3=3.2\times10^4$. For a positive base, $a^{(m/n)}=(\text{the nth root of } a)^m$. Thus $27^{(2/3)}=3^2=9$ and $16^{(-1/2)}=1/4$. To estimate $\sqrt{20}$ without a calculator, $4^2<20<5^2$ gives $4<\sqrt{20}<5$; testing $4.5^2=20.25$ shows $\sqrt{20}$ is just below 4.5. Do not use a rough estimate as an exact surd answer.

Index laws do not turn a sum into a single power: $2^3+2^4=24$, not 2^7. Do not round a surd when an exact answer is requested.

Check powers of ten against the original quantity. Use surds for exact geometry, and round only the final length when the question asks for a decimal.

$0.000072 = 7.2 \times 10^{-5}$

7.2 is the coefficient; -5 is the power

The coefficient is between 1 and 10

Original native-lesson illustration; labels belong to its worked example.

2 Practice

2.1 (Foundation) Evaluate $2^3 \times 2^4$.

2.2 (Foundation) Evaluate 2^3.

2.3 (Foundation) Write 7.2×10^(-5) as a decimal.

2.4 (Higher) Evaluate 16^(3/4).

2.5 (Higher) Evaluate 2^(-3).

3 Exam-style questions

3.1 Decide whether this claim is true. Justify with a concrete calculation or counterexample: For every positive a, a^2+a^3 equals a^5.