5.2 Tree diagrams and sets (Higher)

Name: _____ Class: _____ Date: _____ **Total: 26 marks**

KEY WORDS tree diagram • conditional probability

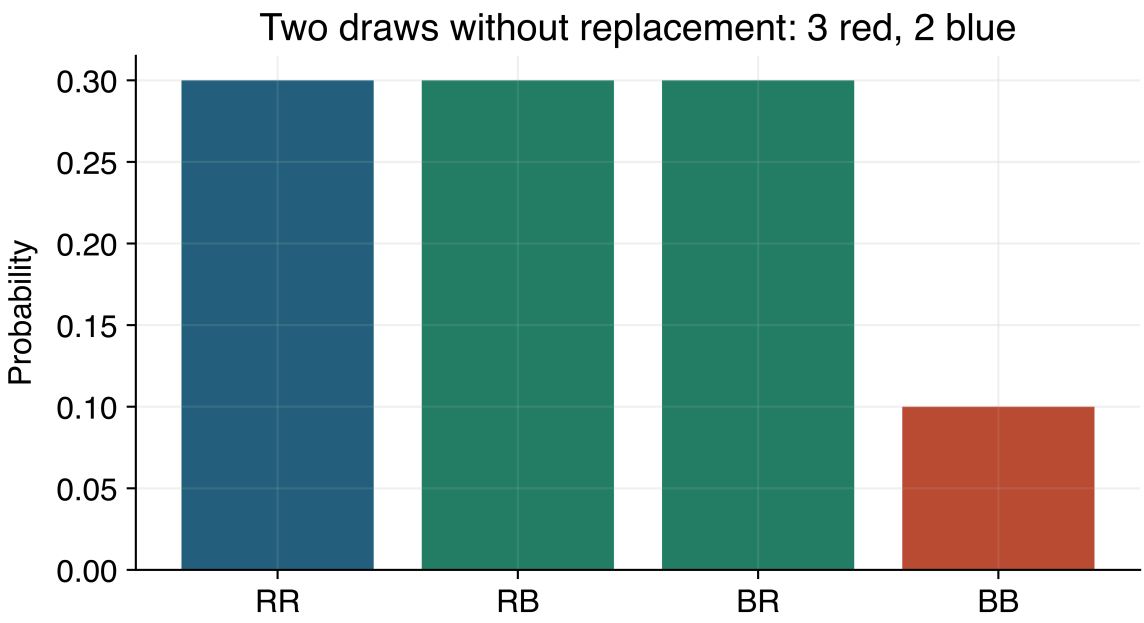
Tree diagrams and sets (Higher)

Objective

Use tree diagrams for independent and dependent combined events, construct possibility spaces, and calculate and interpret conditional probabilities with two-way tables, trees and Venn diagrams (rows P7–P9).

1 Key knowledge and worked examples

Trees and conditioning. Multiply along a path, add disjoint paths. With replacement keeps every denominator fixed (independence); without it, the second-stage denominators shrink. Conditional probability $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$: the denominator is the GIVEN group's total – reversing the condition changes the answer.



Original native-lesson illustration; labels belong to its worked example.

2 Practice

2.1 A bag has 3 red and 2 blue counters. One is drawn, REPLACED, and another drawn. Find $P(\text{two red})$. [3]

2.2 The same bag, now WITHOUT replacement. Find $P(\text{two red})$. [3]

2.3 Without replacement, find $P(\text{one of each colour})$. [3]

2.4 *(June 2024 2H Q4(a), adapted.)* A fair spinner has six equal sections labelled 5, 6, 7 or 8, each appearing at least once, and $P(\text{even}) = P(7)$. Work out $P(5)$. [2]

2.5 In a class of 30, 18 cycle, 12 swim and 7 do both. Find $P(\text{swims} \mid \text{cycles})$ and $P(\text{cycles} \mid \text{swims})$. [3]

3 Exam-style questions

3.1 *(June 2024 3H Q12(b), adapted.)* A machine fails on a given day with probability 0.05, independently each day. Find the probability that it fails on at least one of two days. [3]

3.2 *(Original, expected frequencies.)* A condition affects 20% of a population of 1000. A test is positive for 90% of those with the condition and for 10% of those without it. Work out the probability that a person chosen at random from those testing positive has the condition. [4]

3.3 *(Original.)* $P(A) = 0.4$, $P(B) = 0.3$ and $P(A \cap B) = 0.12$. Are A and B independent? Show working. [3]

3.4 *(Original.)* Two fair dice are rolled. Find $P(\text{total is at least } 11)$. [2]

Solutions

2.1 $\frac{3}{5} \times \frac{3}{5} = \frac{9}{25}$ – replacement keeps both denominators at 5.

2.2 $\frac{3}{5} \times \frac{2}{4} = \frac{6}{20} = \frac{3}{10}$.

2.3 $\frac{3}{5} \times \frac{2}{4} + \frac{2}{5} \times \frac{3}{4} = \frac{6}{20} + \frac{6}{20} = \frac{12}{20} = \frac{3}{5}$ – the two disjoint paths add.

2.4 Six sections with each number at least once and as many 7s as even numbers (the 6s and 8s): sections 5, 5, 6, 7, 7, 8 work, so $P(5) = \frac{2}{6} = \frac{1}{3}$.

2.5 $P(\text{swims} \mid \text{cycles}) = \frac{7}{18}$ (denominator 18 cyclists); $P(\text{cycles} \mid \text{swims}) = \frac{7}{12}$ (denominator 12 swimmers).

3.1 $P(\text{no failure in two days}) = 0.95 \times 0.95 = 0.9025$, so $P(\text{at least one}) = 1 - 0.9025 = 0.0975$.

3.2 Expected counts: with the condition 200, of whom $0.9 \times 200 = 180$ test positive; without it 800, of whom $0.1 \times 800 = 80$ test positive. Positives = 260, so $P(\text{condition} \mid \text{positive}) = \frac{180}{260} = \frac{9}{13} \approx 0.692$.

3.3 $P(A) \times P(B) = 0.4 \times 0.3 = 0.12 = P(A \cap B)$, so A and B are independent.

3.4 Three of the 36 ordered pairs give at least 11 – (5,6), (6,5), (6,6) – so $P = \frac{3}{36} = \frac{1}{12}$.

Local review guidance: tree branches labelled before multiplying, second stage adjusted when there is no replacement; the complement route $1 - 0.95^2$ accepted in 3.1; expected frequencies kept whole until the final ratio – the restricted denominator 260, never 1000. This is authored classroom guidance.