4.5 Properties, congruence and similarity

Name: _____ Class: _____ Date: _____ **Total: 23 marks**

KEY WORDS congruence • sector • midpoint

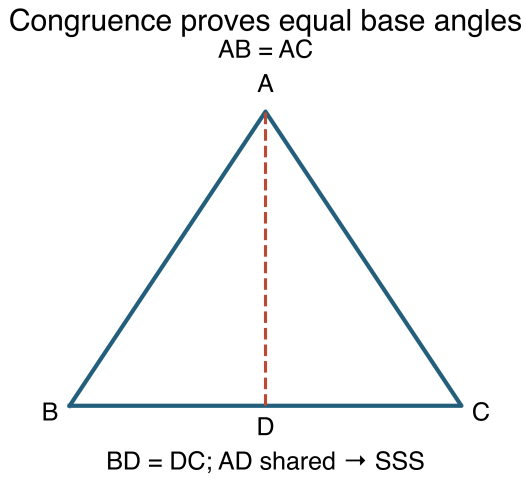
Properties, congruence and similarity

Objective

Derive and apply the properties of special quadrilaterals and triangles, apply the congruence criteria SSS, SAS, ASA and RHS in reasoning and simple proofs, and calculate arc lengths, angles and areas of sectors (rows G4–G5, G18).

1 Key knowledge and worked examples

Properties, congruence and sectors. A rhombus has four equal sides with perpendicular bisecting diagonals; a kite has two pairs of adjacent equal sides; a trapezium has one pair of parallel sides. Triangles are congruent by SSS, SAS, ASA or RHS – the angle in SAS and ASA must be correctly placed, and AAA gives similarity, not congruence. Arc $= \frac{\theta}{360^\circ} \times 2\pi r$; sector area $= \frac{\theta}{360^\circ} \times \pi r^2$; both fractions rearrange to recover θ.



Original native-lesson illustration; labels belong to its worked example.

2 Practice

2.1 A quadrilateral has four equal sides and its diagonals bisect each other at right angles, but the diagonals are not equal in length. Name the quadrilateral. [2]

2.2 Write down the defining side property of a kite. [2]

2.3 Two triangles have three pairs of equal corresponding sides. Which congruence criterion applies? [2]

2.4 Explain why SSA is not a congruence criterion. [2]

2.5 A sector of radius 6 cm has angle 120°. Find its area in terms of π. [2]

2.6 A sector of radius 6 cm has area 9π cm^2. Find its angle. [3]

3 Exam-style questions

3.1 *(June 2024 2H Q12, adapted.)* Two triangles have three pairs of equal corresponding sides. Write down the one-word congruence reason. [1]

3.2 *(Original, proof.)* Triangle ABC is isosceles with $AB = AC$, and D is the midpoint of BC. Prove that the base angles ABC and BCA are equal. [4]

3.3 *(Original.)* A sector of radius 6 cm has angle 90°. Find its arc length and its perimeter, in terms of π. [3]

3.4 *(Original.)* An annulus has outer radius 5 cm and inner radius 3 cm. Find its area in terms of π. [2]

Solutions

2.1 A rhombus (a square would additionally need equal diagonals and right angles).

2.2 Two pairs of adjacent sides equal (adjacent, not opposite).

2.3 SSS.

2.4 Two different triangles can share two sides and a non-included angle – the given data does not fix the triangle, so congruence can fail.

2.5 $\frac{120}{360} \times \pi \times 6^2 = \frac{1}{3} \times 36\pi = 12\pi$ cm^2.

2.6 Fraction $= \dfrac{9\pi}{36\pi} = \frac{1}{4}$, so $\theta = \frac{1}{4} \times 360 = 90°$.

3.1 SSS.

3.2 Triangles ABD and ACD have $AB = AC$ (given), $BD = DC$ (D is the midpoint of BC) and AD shared, so they are congruent (SSS). In congruent triangles, matching angles are equal: $\angle ABD$ matches $\angle ACD$. Since D lies on BC, $\angle ABD = \angle ABC$ and $\angle ACD = \angle BCA$; hence the base angles ABC and BCA are equal.

3.3 Arc $= \frac{90}{360} \times 2\pi \times 6 = 3\pi$ cm; perimeter $= 3\pi + 6 + 6 = 3\pi + 12$ cm.

3.4 $\pi(5^2 - 3^2) = \pi(25 - 9) = 16\pi$ cm^2.

Local review guidance: names with the defining property, not appearance; congruence letters ordered so matching parts correspond; each of the three pairs stated with its reason (given, midpoint, common side) before the criterion; the $\frac{\theta}{360}$ fraction shown for every sector calculation. This is authored classroom guidance.