4.4 Transformations, constructions, loci, circles and vectors

Name: _____ Class: _____ Date: _____ **Total: 27 marks**

KEY WORDS locus • invariant • cyclic quadrilateral

Transformations, constructions, loci, circles and vectors

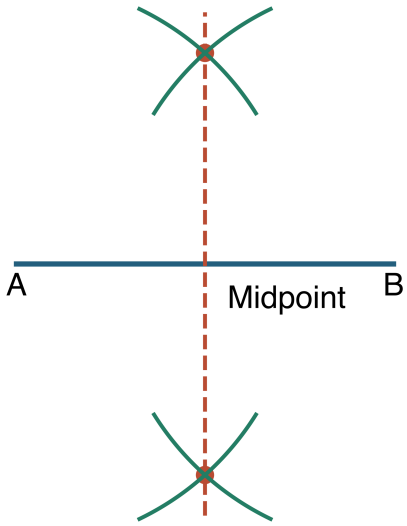
Objective

Identify and construct transformations including negative scale factors, describe invariance under combinations, use constructions and loci, apply circle theorems and use vector methods in proof (rows G7–G10, G25).

1 Key knowledge and worked examples

Transformations, circles and vectors. Enlargement image of P about centre C: $C+k(P-C)$; a negative k places the image on the opposite side. Isometries (rotation, reflection, translation) preserve lengths and angles; two reflections in parallel lines give a translation. Circle: centre angle twice the circumference angle on the same arc; cyclic quadrilateral opposites sum to 180°; tangent $\perp$ radius. Vectors: add components, multiply by scalars; $\overrightarrow{PQ} = \mathbf{q} - \mathbf{p}$; a scalar multiple plus a shared point proves collinearity.

Equal compass radii: join the two arc intersections



Original native-lesson illustration; labels belong to its worked example.

2 Practice

2.1 Reflect the point $(3, 2)$ in the y-axis. [2]

2.2 Rotate the point $(3, 2)$ by $90°$ anticlockwise about the origin. [2]

2.3 *(Higher.)* Enlarge $P = (3, 2)$ about $C = (1, 1)$ with scale factor 2, and again with scale factor -2. [3]

2.4 An arc subtends $100°$ at the centre. Find its angle at the circumference on the same arc. [2]

2.5 One angle of a cyclic quadrilateral is 112°. Find its opposite angle. [2]

2.6 With $\mathbf{a} = (4, 1)$ and $\mathbf{b} = (1, 3)$, find $\mathbf{a} + \mathbf{b}$ and $2\mathbf{a} - \mathbf{b}$. [3]

3 Exam-style questions

3.1 *(June 2024 2H Q1, adapted.)* A circle has centre O. Name in one word each: the region between a chord and its arc; the region between two radii and their arc. [2]

3.2 *(June 2024 1H Q22, adapted.)* Two tangents from an external point P touch a circle, centre O, at A and B. Angle $APB = 24°$. The point C lies on the major arc AB. Work out angle ACB. [4]

3.3 *(June 2024 2H Q23, adapted.)* D, E and F are points with $\overrightarrow{CD} = 6\mathbf{a} + \mathbf{b}$, $\overrightarrow{CE} = 14\mathbf{a} - 3\mathbf{b}$ and $\overrightarrow{CF} = 16\mathbf{a} - 4\mathbf{b}$. (a) Find $\overrightarrow{DE}$ and $\overrightarrow{EF}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. (b) Prove that DEF is a straight line. [4]

3.4 *(Original, loci.)* A and B are 6 cm apart. Describe the locus of points equidistant from A and B; a point on it is 5 cm from A – find its perpendicular distance from AB. [3]

Solutions

2.1 $(-3, 2)$.

2.2 $(-2, 3)$.

2.3 $k = 2$: $C + 2(P - C) = (1, 1) + 2(2, 1) = (5, 3)$. $k = -2$: $(1, 1) - 2(2, 1) = (-3, -1)$, opposite side of the centre.

2.4 $100 \div 2 = 50°$ – angle at the centre is twice the angle at the circumference.

2.5 $180 - 112 = 68°$ – opposite angles of a cyclic quadrilateral sum to $180°$.

2.6 $\mathbf{a} + \mathbf{b} = (5, 4)$; $2\mathbf{a} - \mathbf{b} = (8, 2) - (1, 3) = (7, -1)$.

3.1 The chord-and-arc region is the **segment**; the radii-and-arc region is the **sector**.

3.2 Each tangent is perpendicular to its radius at the contact point, so quadrilateral $OAPB$ has two $90°$ angles: $\angle AOB = 360 - 90 - 90 - 24 = 156°$. The angle at the centre is twice the angle at the circumference on the same arc: $\angle ACB = 156 \div 2 = 78°$.

3.3 (a) $\overrightarrow{DE} = (14\mathbf{a} - 3\mathbf{b}) - (6\mathbf{a} + \mathbf{b}) = 8\mathbf{a} - 4\mathbf{b}$; $\overrightarrow{EF} = (16\mathbf{a} - 4\mathbf{b}) - (14\mathbf{a} - 3\mathbf{b}) = 2\mathbf{a} - \mathbf{b}$. (b) $\overrightarrow{DE} = 4(2\mathbf{a} - \mathbf{b}) = 4\,\overrightarrow{EF}$, so DE is parallel to EF; they share the point E, so DEF is a straight line.

3.4 The perpendicular bisector of AB; half of AB is 3 cm, so the distance is $\sqrt{5^2 - 3^2} = 4$ cm (a 3-4-5 triangle).

Local review guidance: full transformation descriptions – centre, factor, line, angle and direction; the quadrilateral angle sum $360-90-90-24$ then halving for the circumference angle in 3.2; $\overrightarrow{DE} = 4\overrightarrow{EF}$ stated WITH the shared point for the collinearity claim; locus named as a construction, not "the middle". This is authored classroom guidance.