4.3 Pythagoras, trigonometry and vectors

Name: _____ Class: _____ Date: _____ **Total: 28 marks**

KEY WORDS hypotenuse • exact value • collinear

Pythagoras, trigonometry and vectors

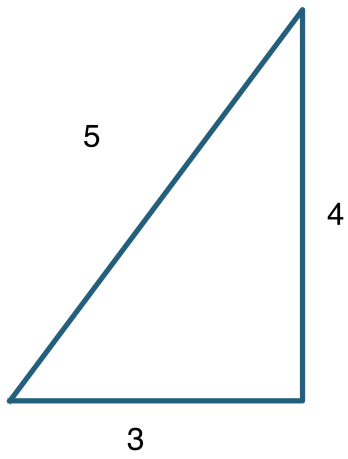
Objective

Apply Pythagoras' theorem and the trigonometric ratios in right-angled triangles in 2D and 3D, use exact trig values, apply the sine and cosine rules and $\frac{1}{2}ab\sin C$, and describe translations as vectors (rows G6, G20–G24).

1 Key knowledge and worked examples

Triangles and vectors. $a^2 + b^2 = c^2$; $\sin\theta = \frac{\text{opp}}{\text{hyp}}$, $\cos\theta = \frac{\text{adj}}{\text{hyp}}$, $\tan\theta = \frac{\text{opp}}{\text{adj}}$; exact values from the $1, \sqrt{3}, 2$ and $1, 1, \sqrt{2}$ triangles. Sine rule $\frac{a}{\sin A} = \frac{b}{\sin B}$; cosine rule $a^2 = b^2 + c^2 - 2bc\cos A$; area $\frac{1}{2}ab\sin C$. A translation by column vector $\binom{x}{y}$ adds x and y to the coordinates; its length is $\sqrt{x^2 + y^2}$.

Label the known sides before choosing a rule



Original native-lesson illustration; labels belong to its worked example.

2 Practice

2.1 A ladder footed 3 m from a wall reaches 4 m up it. Find the ladder's length. [2]

2.2 In a right triangle the opposite and adjacent sides of angle θ are 4 and 3. Find θ to 1 decimal place. [3]

2.3 Write down the exact value of $\sin 30° + \cos 60°$. [2]

2.4 A right triangle has hypotenuse 10 and an angle of 30°. Write down the exact lengths of the two other sides. [3]

2.5 *(Higher.)* Two sides of a triangle, 6 and 8, enclose 60°. Find the third side in surd form. [3]

2.6 *(Higher.)* In a triangle, $a = 4$ is opposite $A = 30°$ and $B = 45°$. Find b in surd form. [3]

3 Exam-style questions

3.1 *(June 2024 2H Q18, adapted.)* In a right triangle the hypotenuse is 23.7 and x is the side adjacent to the marked angle 52°. Work out x to 1 decimal place. [3]

3.2 *(June 2024 1H Q25, adapted.)* Show that the value of $6\sin 30° + 2\cos 30° \times 4\tan 30°$ is an integer. [3]

3.3 *(Original, Higher 3D.)* A cuboid has width 3 cm, depth 4 cm and height 12 cm. Work out the length of its space diagonal. [4]

3.4 *(Original.)* A translation maps $A(1, 2)$ onto $B(5, 5)$. Write the translation as a column vector and find its length. [2]

Solutions

2.1 $c = \sqrt{3^2 + 4^2} = 5$ m.

2.2 $\tan\theta = \frac{4}{3}$, so $\theta = 53.1^\circ$ (1 dp).

2.3 $\frac{1}{2} + \frac{1}{2} = 1$.

2.4 Opposite $= 10\sin 30^\circ = 5$; adjacent $= 10\cos 30^\circ = 5\sqrt{3}$.

2.5 $c^2 = 36 + 64 - 2(6)(8)\cos 60^\circ = 100 - 48 = 52$, so $c = 2\sqrt{13}$.

2.6 $b = \dfrac{4\sin 45^\circ}{\sin 30^\circ} = \dfrac{4 \times \frac{\sqrt{2}}{2}}{\frac{1}{2}} = 4\sqrt{2}$.

3.1 $\cos 52^\circ = \frac{x}{23.7}$, so $x = 23.7\cos 52^\circ = 14.6$ (1 dp).

3.2 $6 \times \frac{1}{2} + \sqrt{3} \times \frac{4}{\sqrt{3}} = 3 + 4 = 7$ – the surds cancel, so the value is the integer 7.

3.3 Base diagonal $= \sqrt{3^2 + 4^2} = 5$; space diagonal $= \sqrt{5^2 + 12^2} = 13$ cm (equivalently $\sqrt{9 + 16 + 144} = 13$).

3.4 $\binom{4}{3}$; length $= \sqrt{4^2 + 3^2} = 5$.

Local review guidance: the correct ratio or rule chosen and named before substitution; surd or exact-π-style forms kept exact where asked; the two-stage diagonal, not $\sqrt{3 + 4 + 12}$; column-vector order x above y. This is authored classroom guidance.