4.2 Perimeter, area and volume, including circles

Name: _____ Class: _____ Date: _____ **Total: 27 marks**

KEY WORDS circumference • frustum • scale factor

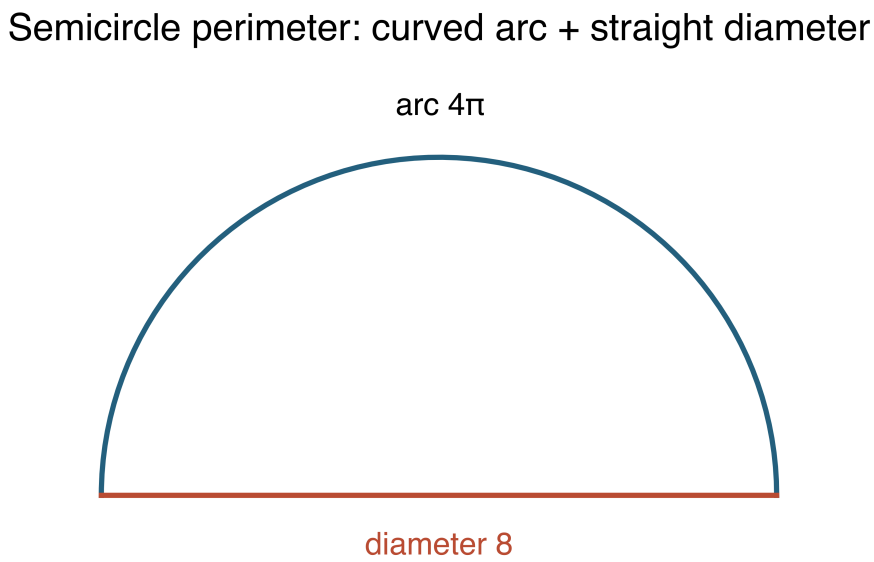
Perimeter, area and volume, including circles

Objective

Calculate perimeters, areas and volumes of plane shapes, circles, sectors, prisms, cylinders, cones and spheres, and use the $k : k^2 : k^3$ similarity ladder for lengths, areas and volumes (rows G14–G17, G19).

1 Key knowledge and worked examples

Mensuration and similarity. Triangle $\frac{1}{2}bh$, parallelogram bh, trapezium $\frac{1}{2}(a+b)h$; circle $C = 2\pi r$, $A = \pi r^2$; arc and sector scale by $\frac{\theta}{360^\circ}$; prism volume = cross-section × length, cylinder $\pi r^2 h$, cone $\frac{1}{3}\pi r^2 h$, sphere $\frac{4}{3}\pi r^3$. Similar figures: lengths k, areas k^2, volumes k^3 – find k from a linear pair before scaling.



Original native-lesson illustration; labels belong to its worked example.

2 Practice

2.1 A trapezium has parallel sides 6 cm and 10 cm with perpendicular height 4 cm. Find its area. [2]

2.2 A triangle has base 8 cm and perpendicular height 5 cm. Find its area. [2]

2.3 A circle has radius 4 cm. Write down its circumference and area in terms of π. [3]

2.4 A cylinder has radius 3 cm and height 5 cm. Find its volume in terms of π. [3]

2.5 Find the perimeter of a semicircle of radius 4 cm, in terms of π. [3]

2.6 A cone has radius 3 cm and perpendicular height 4 cm. Find its slant height and its volume in terms of π. [2]

3 Exam-style questions

3.1 *(June 2024 1H Q12, adapted.)* A sector of a circle has radius 12 cm and angle 60°. Work out the length of its arc, giving your answer in terms of π. [3]

3.2 *(Original, Higher similarity.)* Two spheres have radii 3 cm and 6 cm. (a) Write down the ratio of their surface areas. (b) The smaller sphere has volume 36π cm^3. Find the volume of the larger sphere. [4]

3.3 *(June 2024 2H Q25, adapted.)* A right-angled triangular prism has edges in the ratio $a : b : c : d = 3 : 4 : 5 : 12$ and volume 1125 cm^3. Work out the total length of all of its edges. [5]

Solutions

2.1 $\frac{1}{2}(6+10) \times 4 = 32$ cm^2.

2.2 $\frac{1}{2} \times 8 \times 5 = 20$ cm^2.

2.3 $C = 2\pi(4) = 8\pi$ cm; $A = \pi(4)^2 = 16\pi$ cm^2.

2.4 $\pi \times 3^2 \times 5 = 45\pi$ cm^3.

2.5 Half circumference $= 4\pi$ plus diameter 8: perimeter $= 4\pi + 8$ cm.

2.6 Slant $= \sqrt{3^2+4^2} = 5$ cm; volume $= \frac{1}{3}\pi(3)^2(4) = 12\pi$ cm^3.

3.1 Arc $= \frac{60}{360} \times 2 \times 12 \times \pi = \frac{1}{6} \times 24\pi = 4\pi$ cm.

3.2 (a) Length factor $k = 2$, so surface areas scale by k^2: ratio $9 : 36 = 1 : 4$. (b) Volumes scale by $k^3 = 8$: $36\pi \times 8 = 288\pi$ cm^3.

3.3 Write the edges as $3k, 4k, 5k, 12k$: volume $= \frac{1}{2}(3k)(4k)(12k) = 72k^3 = 1125$, so $k^3 = 15.625$ and $k = 2.5$. Total edge length $= 2(3+4+5)k + 3 \times 12k = 60k = 150$ cm – two of each cross-section edge plus three length edges.

Local review guidance: the $\frac{\theta}{360}$ fraction or the $k : k^2 : k^3$ ladder stated before multiplying; exact π forms where asked, squared or cubed units correct; $72k^3 = 1125$ then cube-rooted. This is authored classroom guidance.