AQA · GCSE · MATHEMATICS • EXERCISE SHEET

3.3 Proportion, rates and gradients

Name: _____ Class: _____ Date: _____ **Total: 26 marks**

KEY WORDS direct proportion • inverse proportion

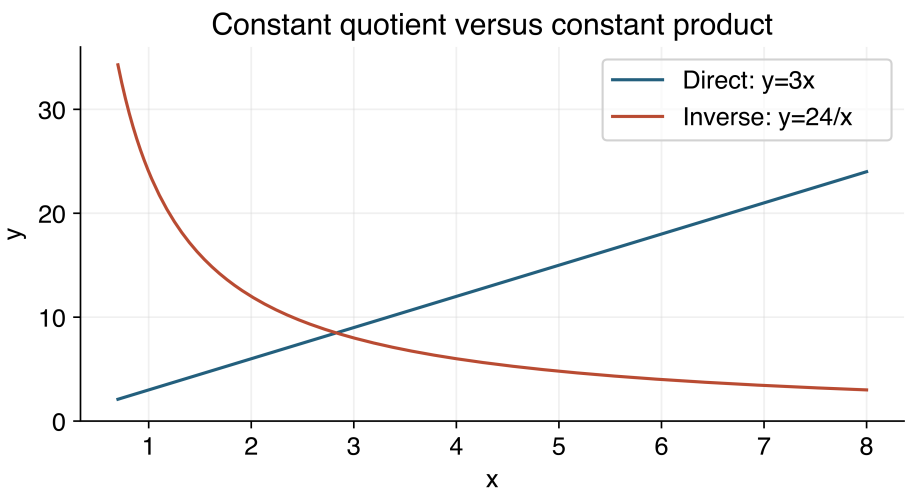
Proportion, rates and gradients

Objective

Solve direct and inverse proportion problems graphically and algebraically, construct proportion equations (Higher), use compound-unit rates, and interpret straight-line and tangent gradients as rates of change (rows R10, R11, R13, R14, R15).

1 Key knowledge and worked examples

Proportion. Direct $y = kx^n$; inverse $y = k/x^n$: find k from one pair. If $y \propto x^3$ with $y = 24$ at $x = 2$, $k = 3$; if $p \propto \frac{1}{x^2}$ with $p = 8$ at $x = 2$, $k = 32$ and $p = \frac{32}{x^2}$. Direct graphs are lines through the origin; inverse graphs are reciprocal-shaped.



Original native-lesson illustration; labels belong to its worked example.

2 Practice

2.1 $y \propto x$ with $y = 21$ at $x = 7$: find the equation and y at $x = 12$ [3]

2.2 $y \propto x^2$ with $y = 48$ at $x = 4$: find y at $x = 7$ [3]

2.3 p is inversely proportional to x; $p = 8$ when $x = 6$. Find p when $x = 8$. [3]

2.4 A job pays £9.20 per hour. Find the pay for 7.5 hours. [2]

2.5 On a distance-time graph a line rises 90 units of distance per 1.3 units of time. What rate does the gradient give, and what is its value to 3 s.f.? [3]

3 Exam-style questions

3.1 *(Original, Higher construct.)* y is directly proportional to the square of x. When $x = 3$, $y = 36$. (a) Find the equation linking x and y. (b) Find y when $x = 5$. (c) Find x when $y = 100$. [5]

3.2 *(Original, Higher inverse construction.)* T is inversely proportional to the square root of m. $T = 6$ when $m = 4$. (a) Find the equation. (b) Find T when $m = 36$. [4]

3.3 *(Original, Higher tangent rate.)* A curve passes through the point where $x = 2$; the tangent there rises 5 units of y per 0.4 units of x. State the instantaneous rate of change at $x = 2$ and explain how it differs from the average rate over an interval. [3]

Solutions

2.1 $k = 3$, so $y = 3x$; $y = 36$.

2.2 $k = 3$; $y = 3 \times 49 = 147$.

2.3 $k = 48$; $p = 48 \div 8 = 6$.

2.4 $9.2 \times 7.5 =$ £69.

2.5 The gradient is the speed: $90 \div 1.3 = 69.2$ (3 s.f.).

3.1 (a) $36 = 9k$, so $k = 4$ and $y = 4x^2$. (b) $y = 100$. (c) $4x^2 = 100$ gives $x = 5$.

3.2 (a) $T = \frac{k}{\sqrt{m}}$: $6 = \frac{k}{2}$, so $k = 12$ and $T = \frac{12}{\sqrt{m}}$. (b) $T = \frac{12}{6} = 2$.

3.3 Rate $= 5 \div 0.4 = 12.5$. The tangent gives the INSTANTANEOUS rate at the single point; the average rate uses a chord joining the interval's end points and can differ on a curve.

Local review guidance: k found from one pair before use; exact values; tangent vs chord distinction stated in words. This is authored classroom guidance.