

3.2.1 Scale, similarity and growth (cut of 3.2)

Name: _____ Class: _____ Date: _____

Total: 29 marks

KEY WORDS compound interest • area scale factor

Scale, similarity and growth

Objective

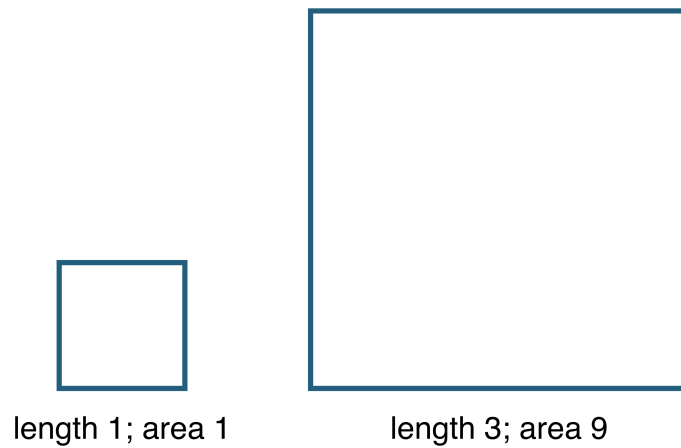
Use scale factors and maps, the $k : k^2 : k^3$ similarity ladder, compound growth and decay including general iterative processes (rows R2, R12, R16).

1 Key knowledge and worked examples

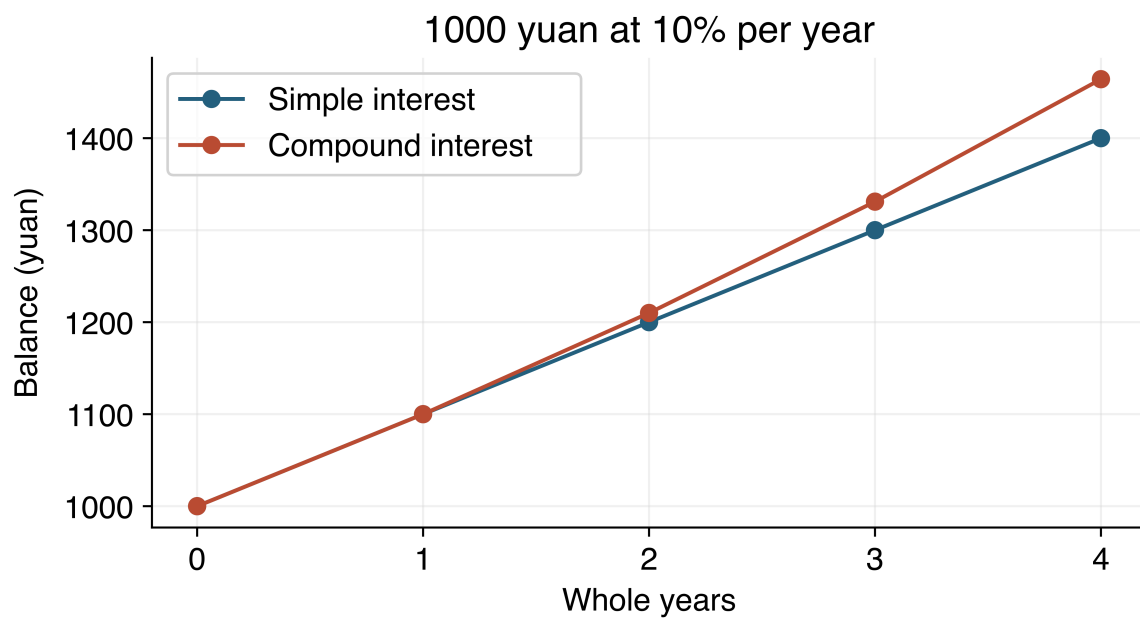
Similarity ladder. Lengths k : areas k^2 : volumes k^3 . Statues of heights 3 : 5 have volumes 27 : 125: a 216 cm³ smaller statue scales to $216 \times \frac{125}{27} = 1000$ cm³. Maps and scale drawings convert through the stated ratio.

Growth and decay. $A = P(1 \pm \frac{r}{100})^n$: £2000 at 3% for 4 years gives $2000 \times 1.03^4 = £2251.02$. Sequential changes multiply: $90000 \times 1.03 \times 1.08 = 100116$ (the 2H June 2024 Q10 anchor). Iterative processes at Higher: $x_{n+1} = 1.05x_n$; the limit solves $x = f(x)$.

A length factor of 3 acts in both area dimensions



Original native-lesson illustration; labels belong to its worked example.



Original native-lesson illustration; labels belong to its worked example.

2 Practice

2.1 Similar jugs have heights 8 cm and 12 cm. The smaller holds 144 cm^3 . Find the larger's volume. [3]

2.2 Similar photographs have areas 18 cm^2 and 50 cm^2 . Find the ratio of their widths. [3]

2.3 On a 1 : 25 000 map a road measures 6 cm. Find the real length in km. [2]

2.4 £2000 is invested at 3% compound interest. Find the value after 4 years and the interest earned. [3]

2.5 A car worth £12000 depreciates 15% per year. Find its value after 3 years. [2]

2.6 With $x_{n+1} = 1.05x_n$ from $x_1 = 800$, find x_3 and state (with a reason) whether the sequence approaches a finite limit. [3]

3 Exam-style questions

3.1 (*Edexcel-style, 2H June 2024 Q10 adapted.*) A house is worth £90000. Its value rises by 3% one year and by 8% the next. (a) Show its value is now £100116. (b) A second house must exceed £100000 after the same changes. Did it start above or below £89000? Justify. [4]

3.2 (*Original, Higher ladder.*) Two similar bottles have heights in the ratio 2 : 5. The larger holds 500 cm³. Find the smaller's volume. (b) The smaller's label area is 30 cm². Find the larger's label area. [5]

3.3 (*Original, Higher iterative decay.*) Each hour a quantity keeps 80% of its value, with $x_1 = 1000$ and $x_{n+1} = 0.8x_n$. (a) Find x_3 . (b) Explain why the values approach zero but never reach it. [4]

Solutions

2.1 $k = \frac{12}{8} = \frac{3}{2}$; volumes scale by $\frac{27}{8}$: $144 \times \frac{27}{8} = 486 \text{ cm}^3$.

2.2 $\sqrt{18} : \sqrt{50} = 3 : 5$.

2.3 $6 \times 25\,000 = 150\,000 \text{ cm} = 1.5 \text{ km}$.

2.4 $2000 \times 1.03^4 = \text{£}2251.02$; interest $\text{£}251.02$.

2.5 $12000 \times 0.85^3 = \text{£}7369.50$.

2.6 $x_2 = 840$, $x_3 = 882$; no finite limit – each term is 1.05 times the last, so values grow without bound.

3.1 (a) $90000 \times 1.03 \times 1.08 = \text{£}100116$. (b) $100000 \div (1.03 \times 1.08) = \text{£}89896$ to the nearest pound, so the second house started ABOVE $\text{£}89000$ (it needed at least $\text{£}89896$).

3.2 (a) $k = \frac{2}{5}$; volume factor $\frac{8}{125}$: $500 \times \frac{8}{125} = 32 \text{ cm}^3$. (b) Area factor $\frac{4}{25}$: $30 \div \frac{4}{25} = 187.5 \text{ cm}^2$.

3.3 (a) $x_2 = 800$, $x_3 = 640$. (b) Each term is 0.8 times the previous, so values shrink toward zero; but $0.8x_n$ is never exactly 0 for any finite n – zero is the limit, not a term.

Local review guidance: the ladder direction stated before multiplying; exact values; iterative answers justify the limit claim. This is authored classroom guidance.