2.5 Sequences

Name: _____ Class: _____ Date: _____ **Total: 25 marks**

KEY WORDS nth term

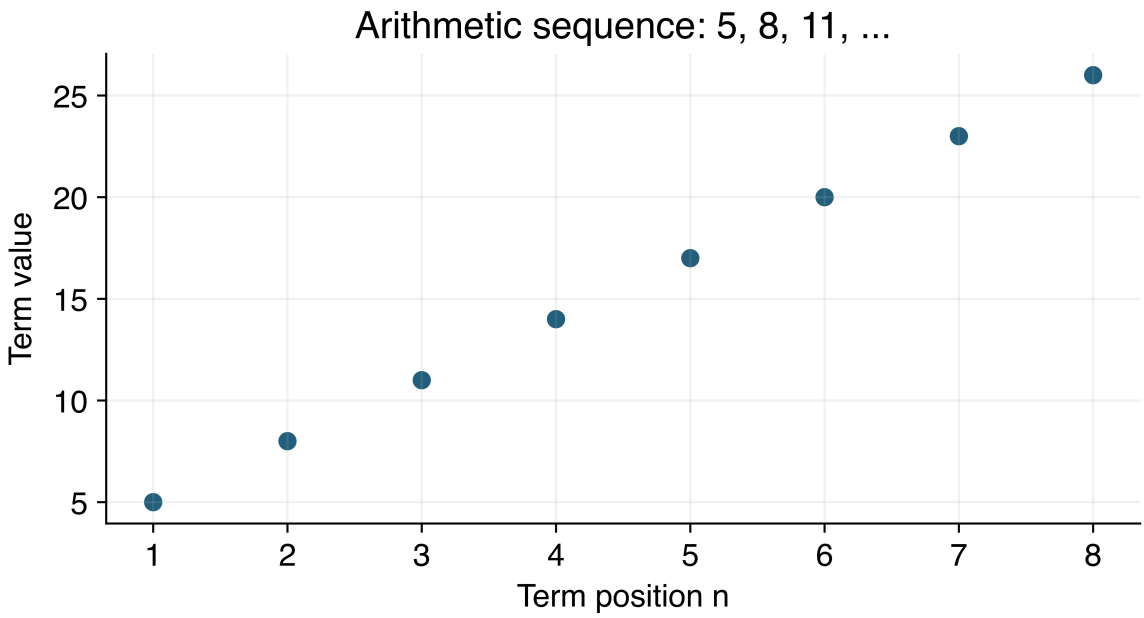
Sequences

Objective

Generate terms from term-to-term and position-to-term rules; recognise triangular, square, cube, Fibonacci, quadratic and geometric progressions incl. surd ratios; deduce linear and quadratic nth terms (rows A23–A25).

1 Key knowledge and worked examples

Types and nth terms. Triangular $\frac{1}{2}n(n+1)$, squares, cubes; Fibonacci adds the two previous terms. Linear nth term $an+b$; quadratic an^2+bn+c with $a=\frac{1}{2}$(second difference): for $5, 9, 17, 29, 45, \ldots$ the differences are $4, 8, 12, 16$ (second difference 4), so $a=2$ and the sequence is $2n^2-2n+5$ – check $n=4$: $32-8+5=29$. Geometric: ratio constant.



Original native-lesson illustration; labels belong to its worked example.

2 Practice

2.1 Write the next two terms of $5, 9, 17, 29, 45, \ldots$ [2]

2.2 Find the nth term of $4, 7, 12, 19, 28, \ldots$ [3]

2.3 Find the nth term of $6, 11, 18, 27, 38, \ldots$ [3]

2.4 A Fibonacci-type sequence starts $2, 5, \ldots$: write the next three terms. [2]

2.5 Which term of the sequence with nth term $3n + 2$ equals 62? [2]

3 Exam-style questions

3.1 *(Original.)* A sequence has nth term $n^2 + 2n$. (a) Write down the first four terms. (b) Which term equals 168? [4]

3.2 *(Original.)* The first differences of a sequence are $5, 9, 13, 17, \ldots$. (a) Find the next difference. (b) The first term is 3: write the first four terms. (c) Find the nth term. [5]

3.3 *(Original.)* y is proportional to x^2; $y = 48$ when $x = 4$. (a) Find the equation. (b) Find y when $x = 7$. [4]

Solutions

2.1 Differences $20, 24$ follow the $+4$ pattern: terms $65, 89$.

2.2 Differences $3, 5, 7, 9$; second difference 2, so $a = 1$: n^2 leaves $3, 3, 3, 3$, giving $n^2 + 3$.

2.3 Differences $5, 7, 9, 11$; second difference 2, so $a = 1$: subtracting n^2 (1, 4, 9, 16) leaves $5, 7, 9, 11 = 2n + 3$: $n^2 + 2n + 3$.

2.4 $7, 12, 19$.

2.5 $3n + 2 = 62$ gives $n = 20$.

3.1 (a) $3, 8, 15, 24$. (b) $n^2 + 2n = 168$ gives $(n + 1)^2 = 169$, so $n = 12$.

3.2 (a) Differences increase by 4: next is 21. (b) $3, 8, 17, 30$. (c) Second difference of the terms is 4, so $a = 2$; subtracting $2n^2$ (which gives $2, 8, 18, 32$) leaves $1, 0, -1, -2 = 2 - n$: so the nth term is $2n^2 - n + 2$ – check $n = 4$: $32 - 4 + 2 = 30$. ✓

3.3 (a) $y = kx^2$: $48 = 16k$ gives $k = 3$. (b) $y = 3 \times 49 = 147$.

Local review guidance: second differences before guessing a; nth terms checked on two positions; sequence types named. This is authored classroom guidance.