2.4 Iteration and inequalities

Name: _____ Class: _____ Date: _____ **Total: 25 marks**

KEY WORDS iteration

Iteration and inequalities

Objective

Find approximate solutions by iteration with suffix notation (A20, Higher); solve linear and quadratic inequalities with number-line, set-notation and graphical representation (A22).

1 Key knowledge and worked examples

Iteration. Rearrange the equation to $x = f(x)$, then cycle $x_{n+1} = f(x_n)$ from a starting value, recording terms to a stated accuracy. From $x^3 - 3x - 1 = 0$: $x_{n+1} = \sqrt[3]{3x_n + 1}$ from $x_1 = 1$ gives $1.587, 1.793, 1.855, 1.872, \ldots \to 1.879$ (3 d.p.). Check the limit back in the ORIGINAL equation.

Inequalities. Same moves as equations – multiplying/dividing by a negative flips the sign. Number line: open circle strict, closed circle included. Higher: quadratic inequalities and two-variable regions – $x^2 - x - 6 > 0$ gives $x < -2$ or $x > 3$, i.e. $\{x\colon x < -2\} \cup \{x\colon x > 3\}$; dashed boundary for strict.

2 Practice

2.1 Solve $3x - 2 < 10$ and represent on a number line [2]

2.2 Solve $7 - x \leq 5$ [2]

2.3 Solve $-7 \leq 3x + 2 < 6$ [3]

2.4 Solve $x^2 - x - 6 > 0$ [3]

2.5 With $x_{n+1} = \frac{1}{2}x_n + 3$ from $x_1 = 1$, find x_2, x_3, x_4 and state the limit. [3]

3 Exam-style questions

3.1 *(Original.)* The equation $x^3 - 3x - 1 = 0$ has a root between 1 and 2. (a) Show that rearranging gives $x = \sqrt[3]{3x+1}$. (b) Use the iteration $x_{n+1} = \sqrt[3]{3x_n + 1}$ from $x_1 = 1$ to find x_4. (c) Verify the root is 1.879 to 3 d.p. [5]

3.2 *(Original.)* Solve $x^2 + 3x + 2 > 0$ and write the solution set in set notation. [4]

3.3 *(Original.)* Shade or describe the region satisfying $x \geq 0$, $y \geq 1$ and $x + y \leq 5$, stating the vertices. [3]

Solutions

2.1 $x < 4$: open circle at 4, arrow left.

2.2 $-x \leq -2$, so (flip) $x \geq 2$.

2.3 $-9 \leq 3x < 4$, so $-3 \leq x < \frac{4}{3}$.

2.4 $(x-3)(x+2) > 0$: positive outside the roots, so $x < -2$ or $x > 3$.

2.5 $x_2 = 3.5$, $x_3 = 4.75$, $x_4 = 5.375$; limit $x = 6$ (solves $x = \frac{1}{2}x + 3$).

3.1 (a) $x^3 = 3x + 1$ from the equation, so $x = \sqrt[3]{3x+1}$. (b) $x_2 = 1.587$, $x_3 = 1.793$, $x_4 = 1.855$. (c) Continuing: $x_5 = 1.872$, $x_6 = 1.877$; and $1.8785^3 - 3(1.8785) - 1 < 0 < 1.8795^3 - 3(1.8795) - 1$, so the root rounds to 1.879.

3.2 $(x+1)(x+2) > 0$: $x < -2$ or $x > -1$, i.e. $\{x\colon x < -2\} \cup \{x\colon x > -1\}$.

3.3 Triangle with vertices $(0,1)$, $(0,5)$, $(4,1)$; shade inside.

Local review guidance: rearrangement shown before iterating; iterations to the stated accuracy; inequality notation matches strict/included endpoints. This is authored classroom guidance.