2.2 Functions and graphs

Name: _____ Class: _____ Date: _____ **Total: 25 marks**

KEY WORDS gradient

Functions and graphs

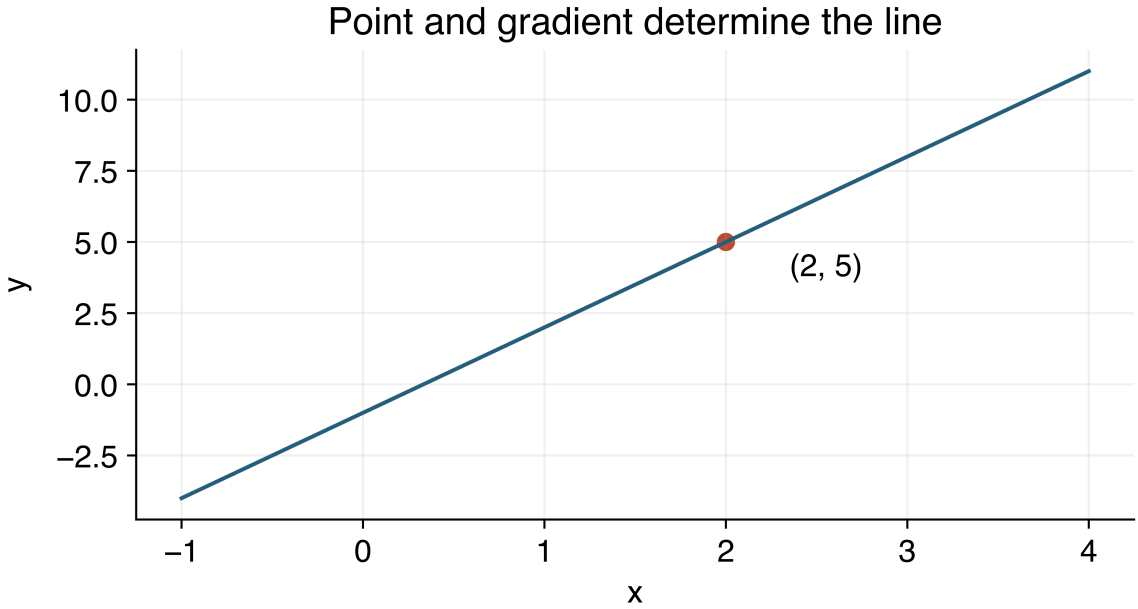
Objective

Use function notation with inverse and composite functions (Higher); work with coordinates, lines through points with gradients incl. perpendicular (Higher); quadratic graphs with turning points by completing the square; cubic, reciprocal, exponential and trig graphs; the circle equation with tangents (rows A7–A16).

1 Key knowledge and worked examples

Functions. $f(x)$ maps each input to one output. Higher: $\mathrm{fg}(x) = f(g(x))$ – apply g first; f^{-1} undoes. For $f(x) = 3x - 5$ and $g(x) = x^2$: $\mathrm{fg}(2) = f(4) = 7$ and $f^{-1}(x) = \frac{x+5}{3}$.

Lines and curves. $m = \frac{y_2-y_1}{x_2-x_1}$; perpendicular $m_1m_2 = -1$. Completing the square: $y = x^2 - 6x + 5 = (x-3)^2 - 4$, minimum $(3, -4)$. Circle: $x^2 + y^2 = r^2$; tangent at (x_1, y_1) has gradient $-x_1/y_1$.



Original native-lesson illustration; labels belong to its worked example.

2 Practice

2.1 For $f(x) = 3x - 5$ and $g(x) = x^2$, find fg(2) [2]

2.2 Find $f^{-1}(x)$ for $f(x) = 3x - 5$ [2]

2.3 Find the equation of the line through $(1, 7)$ and $(2, 9)$ [3]

2.4 Write down the gradient of the line perpendicular to $m = \frac{2}{5}$ [1]

2.5 Write $y = x^2 - 6x + 5$ in the form $(x + a)^2 + b$ and state the minimum point [3]

2.6 The circle $x^2 + y^2 = 25$ passes through $(3, 4)$. Write down the gradient of the tangent there. [2]

3 Exam-style questions

3.1 *(Original.)* A line joins $A(-1, 3)$ to $B(3, 11)$. (a) Find its equation. (b) Find the equation of the perpendicular through the midpoint of AB. [5]

3.2 *(Original.)* Sketch or describe $y = 2^x$, giving the y-intercept and the behaviour as x grows and as $x \to -\infty$. [3]

3.3 *(Original.)* For $y = x^2 + 6x + 2$: (a) write in completed-square form; (b) hence solve $x^2 + 6x + 2 = 0$ giving exact roots. [4]

Solutions

2.1 $g(2) = 4$; $f(4) = 7$.

2.2 $y = 3x - 5 \Rightarrow x = \frac{y+5}{3}$, so $f^{-1}(x) = \frac{x+5}{3}$.

2.3 $m = 2$, so $y = 2x + 5$.

2.4 $-\frac{5}{2}$.

2.5 $(x-3)^2 - 4$; minimum $(3, -4)$.

2.6 Radius gradient $4/3$; tangent $-3/4$.

3.1 (a) $m = \frac{11-3}{3-(-1)} = 2$, so $y = 2x + 5$. (b) Midpoint $(1, 7)$; perpendicular gradient $-\frac{1}{2}$: $y - 7 = -\frac{1}{2}(x - 1)$, so $y = -\frac{1}{2}x + \frac{15}{2}$.

3.2 Passes $(0, 1)$; rises ever faster as x grows; approaches the x-axis (never 0) as $x \to -\infty$.

3.3 (a) $(x+3)^2 - 7$. (b) $(x+3)^2 = 7$, so $x = -3 \pm \sqrt{7}$.

Local review guidance: gradient then point-slope form; exact roots; graph descriptions name intercept AND end behaviour. This is authored classroom guidance.