2.1 Notation, manipulation and proof

Name: _____ Class: _____ Date: _____ **Total: 23 marks**

KEY WORDS identity

Notation, manipulation and proof

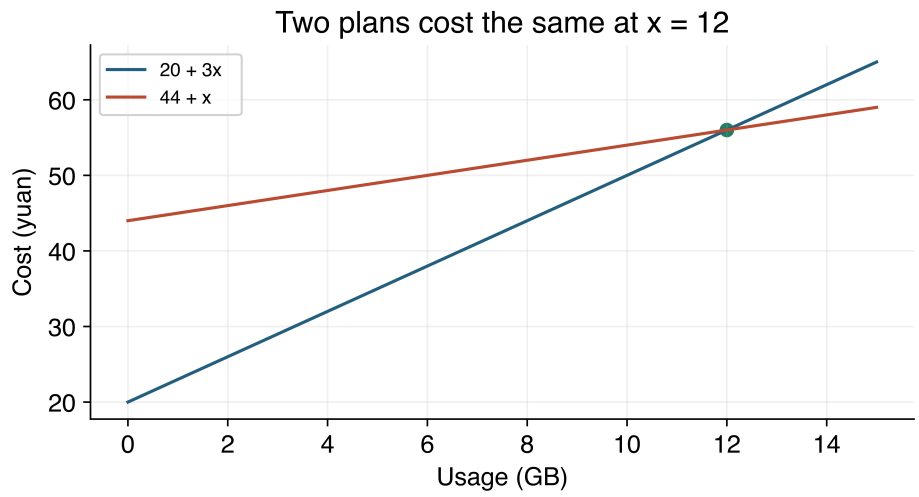
Objective

Use algebraic notation and vocabulary, substitute into formulae, manipulate expressions including surds and algebraic fractions (Higher), change the subject, and argue with identities and proofs (rows A1–A6 across the three tiers).

1 Key knowledge and worked examples

Notation and manipulation. ab, $3y$, a^2, $\frac{a}{b}$; coefficients as fractions not decimals; answers in simplest form. Collect, bracket, factorise, index laws. Higher: surds and algebraic fractions – $(3+\sqrt{2})(1-2\sqrt{2}) = -1-5\sqrt{2}$; $\frac{x^2-9}{x^2+7x+12} = \frac{x-3}{x+4}$.

Identities and proofs. An equation holds for SOME values; an identity for ALL: $(x+2)^2 \equiv x^2+4x+4$. Proofs expand and compare both sides – examples never prove. The June 2024 3H anchor's verdict: "it is true for all values of x".



Original native-lesson illustration; labels belong to its worked example.

2 Practice

2.1 Expand and simplify $3(2x-5)+4(x+1)$ [2]

2.2 Expand and simplify $(x+5)(x-3)$ [2]

2.3 Factorise $x^2-2x-24$ [2]

2.4 Factorise $6x^2-7x-3$ [2]

2.5 Expand and simplify $(3+\sqrt{2})(1-2\sqrt{2})$ [3]

2.6 Make t the subject of $v = u + at$ [2]

3 Exam-style questions

3.1 *(Original, 3H June 2024 Q4 family.)* Show that $(2x+3)^2 - (2x+1)(2x+5) \equiv 4$. [3]

3.2 *(Original, 1H June 2024 Q17 family.)* Make x the subject of $y = \dfrac{3x+7}{x}$. [4]

3.3 *(Original, 1H June 2024 Q9 family.)* Simplify fully $\dfrac{26}{16.4}$, giving your answer as a fraction in its lowest terms. [3]

Solutions

2.1 $6x - 15 + 4x + 4 = 10x - 11$.

2.2 $x^2 + 2x - 15$.

2.3 $(x - 6)(x + 4)$.

2.4 $(3x + 1)(2x - 3)$.

2.5 $3 - 6\sqrt{2} + \sqrt{2} - 4 = -1 - 5\sqrt{2}$.

2.6 $at = v - u$, so $t = \dfrac{v - u}{a}$.

3.1 LHS $= (4x^2 + 12x + 9) - (4x^2 + 12x + 5) = 4 =$ RHS for all x: an identity. ✓

3.2 Multiply by x: $xy = 3x + 7$, so $xy - 3x = 7$, factorise $x(y - 3) = 7$, giving $x = \dfrac{7}{y - 3}$.

3.3 $\dfrac{26}{16.4} = \dfrac{260}{164} = \dfrac{65}{41}$.

Local review guidance: factor before cancelling; simplest forms; identity proofs end with the "all values" statement. This is authored classroom guidance.