Exercise walk-through

6.2.3 Histogram Density

AQA · GCSE · Mathematics

Averages and spread

- The **mean** is the total divided by the number of values.
- The **median** is the middle value when the data is in order.
- The **mode** is the most common value; for grouped data, the **modal class** is the group with the highest frequency.

$$\text{mean} = \frac{\sum fx}{\sum f}$$

Histograms, cumulative frequency and box plots (H)

- **Histograms.** In a **histogram** the area of each bar, not its height, shows the frequency.
- When the **class intervals** have different widths, the height is the **frequency density**:
- **Worked example 6 (H).** Classes $0 \leq x < 10$, $10 \leq x < 20$ and $20 \leq x < 40$ have frequencies 20, 30 and 20.
- The **class width** of the last class is 20, so the frequency densities are $20 \div 10 = 2$, $30 \div 10 = 3$ and $20 \div 20 = 1$.

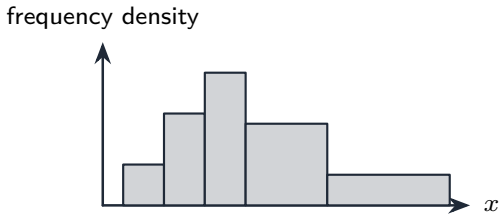
$$\text{frequency density} = \frac{\text{frequency}}{\text{class width}}$$

Common errors (2)

- (H) Drawing **histogram** bars with height equal to frequency when the class widths differ.
- (H) Plotting **cumulative frequency** at the midpoint instead of the upper end of the class.
- Comparing only averages; a comparison also needs a measure of spread.
- Saying that **correlation** proves cause.

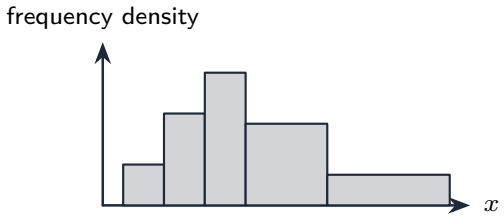
Exercise 2.1

2.1 *(H)* A class $10 < x \leq 20$ has frequency 30. Work out its frequency density. [1]



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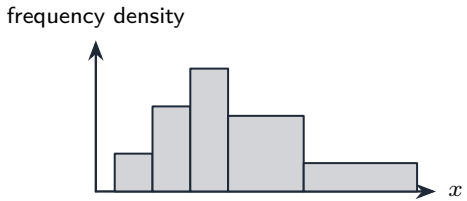


2.1

- $\text{FD} = \dfrac{30}{10} = \mathbf{3}$

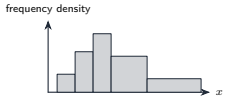
Setup for exercise 2.2

2.2 *(H)* A class $20 < x \leq 40$ has frequency density 1.5. Work out its frequency. [1]



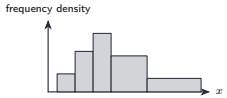
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2.3 *(H)* Explain why a histogram with unequal class widths uses frequency density, not frequency. [1]

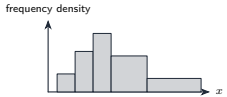


2.4 *(H)* Work out the frequency density for each class. [2]

Time t (min)	$0 < t \leq 5$	$5 < t \leq 10$	$10 < t \leq 20$	$20 < t \leq 40$
Frequency	10	25	30	20

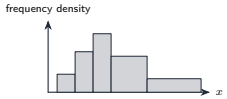


2.5 *(H)* A bar for $40 < x \leq 60$ has frequency 12. Work out the height of the bar. [1]



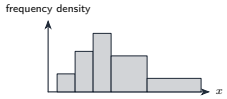
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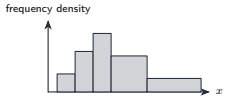


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2.2

- frequency $= 1.5 \times 20 =$ **30**

2.3

Full solution on next slide

Full solution 2.2

2.2

- frequency $= 1.5 \times 20 =$ **30**

2.3

- the area must show the frequency; with unequal widths, plotting frequency would make wide classes look too big

2.4

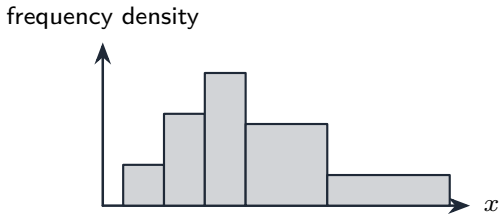
- FD $= \dfrac{\text{frequency}}{\text{width}}$: $\dfrac{10}{5}, \dfrac{25}{5}, \dfrac{30}{10}, \dfrac{20}{20}$
- **2**, **5**, **3**, **1**

2.5

- FD $= \dfrac{12}{20} =$ **0.6**

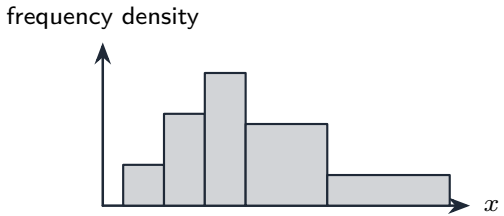
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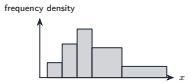


3.1

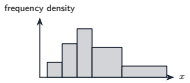
- 15 to 20: $3 \times 5 = 15$; 20 to 30: $1 \times 10 = 10$
- $15 + 10 = \mathbf{25}$

Exercise 3.2

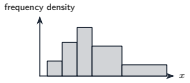
3.2 *(H)* Use the table in **2.4**. Estimate how many times are more than 15 minutes. [2]



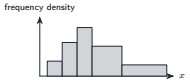
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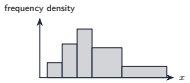


3.5 *(H)* In a histogram, the bar for $20 < x \leq 30$ has height 2.4 and the bar for $30 < x \leq 50$ has height 0.9. The class $20 < x \leq 30$ has frequency 24. Work out the frequency of $30 < x \leq 50$, and explain why comparing the bar heights alone would mislead. [3]

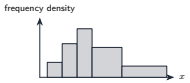


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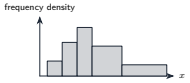
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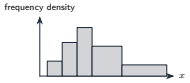
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3.2

- 15 to 20: half of the 30 in $10 < t \leq 20$, so 15; plus all 20 in $20 < t \leq 40$
- $15 + 20 = \mathbf{35}$

3.3

- total 70, so the median is the 35th value; the first class holds 20

Full solution on next slide

Full solution 3.2

3.2

- 15 to 20: half of the 30 in $10 < t \leq 20$, so 15; plus all 20 in $20 < t \leq 40$
- $15 + 20 = \mathbf{35}$

3.3

- total 70, so the median is the 35th value; the first class holds 20
- 15 more are needed from the 30 in 10–20: $\frac{15}{30}$ of the way across
- median $\approx 10 + \frac{15}{30} \times 10 = \mathbf{15}$

3.4

- midpoints 5, 15, 30: $\sum fx = 5 \times 20 + 15 \times 30 + 30 \times 20 = 100 + 450 + 600 = 1150$
- mean $\approx \dfrac{1150}{70} = \mathbf{16.4}$

3.5

- check: $2.4 \times 10 = 24$; second class: frequency $= 0.9 \times 20 = \mathbf{18}$
- the second bar is twice as wide, so its area, not its height, shows the frequency: 18 is only a little less than 24