Exercise walk-through

5.2.2
Conditional Counts

AQA · GCSE · Mathematics

The probability scale and outcomes

- The probability of an event is a number from 0 to 1 on the **probability scale** . 0 means impossible, 1 means certain, and 0.5 means an even chance.
- Write probabilities as fractions, decimals or percentages, never as "1 in 4" or "1 : 4".
- When every **outcome** is **equally likely** , the **theoretical probability** is
- For a fair six-sided die, $P(\text{even}) = \frac{3}{6} = \frac{1}{2}$.

$$P(\text{event}) = \frac{\text{number of outcomes in the event}}{\text{total number of outcomes}}$$

Conditional probability (H)

- A **conditional probability** is the probability of an event when you already know that another event has happened.
- $P(A \mid B)$ means "the probability of A, given B".
- Knowing B shrinks the group you count from to just the members of B:
- **Worked example 9 (H).** In the class of 30 (18 cycle, 12 swim, 7 both), $P(\text{swims} \mid \text{cycles}) = \frac{7}{18}$, because only the 18 cyclists count.

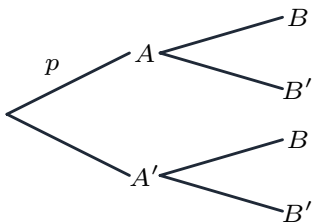
$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$$

Key terms (2)

- **Expected frequency**: the number of times an event should happen on average: trials $\times$ probability.
- **Independent**: events where one does not change the probability of the other.
- **Sample space**: a list or table of all possible outcomes.
- **Conditional probability**: the probability of an event given that another event has happened.

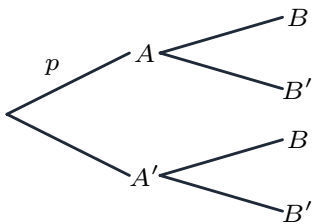
Exercise 2.1

2.1 *(H)* In a class of 30, 18 cycle, 12 swim and 7 do both. Work out $P(\text{swims} \mid \text{cycles})$. [1]



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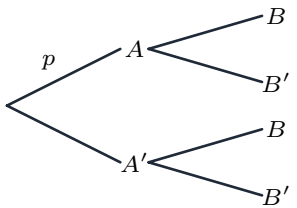


2.1

- only the 18 cyclists count, and 7 of them swim: $\mathbf{\frac{7}{18}}$

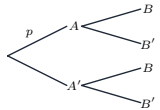
Setup for exercise 2.2

2.2 *(H)* For the class in **2.1**, work out $P(\text{cycles} \mid \text{swims})$. [1]

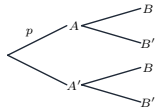


Exercise 2.2

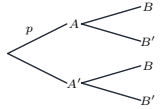
2.3 *(H)* A bag has 5 red and 3 blue counters. Two are taken without replacement. The first is red. Work out the probability that the second is red. [1]



2.4 *(H)* Of 100 students, 40 study art, and 25 of these are girls. 30 of the other 60 students are girls. Work out $P(\text{girl} \mid \text{art})$ and $P(\text{art} \mid \text{girl})$. [2]



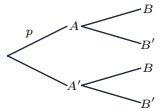
2.5 *(H)* $P(A \cap B) = 0.12$ and $P(B) = 0.3$. Work out $P(A \mid B)$. [2]



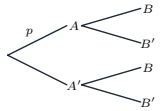
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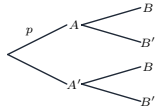
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2.2

- only the 12 swimmers count, and 7 of them cycle: $\mathbf{\frac{7}{12}}$

2.3

Full solution on next slide

Full solution 2.2

2.2

- only the 12 swimmers count, and 7 of them cycle: $\mathbf{\frac{7}{12}}$

2.3

- 4 red are left out of 7: $\mathbf{\frac{4}{7}}$

2.4

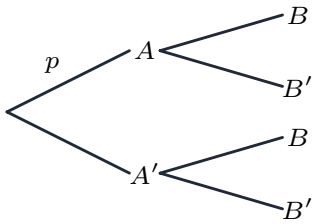
- $P(\text{girl} \mid \text{art}) = \frac{25}{40} = \mathbf{\frac{5}{8}}$
- girls $= 25 + 30 = 55$, so $P(\text{art} \mid \text{girl}) = \frac{25}{55} = \mathbf{\frac{5}{11}}$

2.5

- $P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{0.12}{0.3} = \mathbf{0.4}$

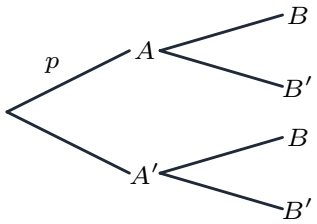
Exercise 3.1

3.1 *(H)* Use **2.1** and **2.2** to explain why $P(A \mid B)$ is not always equal to $P(B \mid A)$. [1]



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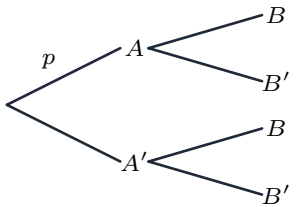


3.1

- both use the 7 who do both, but divide by different totals: 18 cyclists or 12 swimmers, so $\frac{7}{18} \neq \frac{7}{12}$

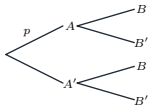
Setup for exercise 3.2

3.2 *(H)* $P(A) = 0.4$, $P(B) = 0.5$ and $P(A \cap B) = 0.2$. Are A and B independent? Show how you decide. [2]

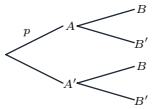


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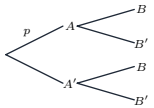
3.3 *(H)* A drawer has 6 black and 4 white socks. Two are taken without replacement. Given that they are the same colour, work out the probability that both are black. [3]



3.4 *(H)* The probability that it rains on a school day is 0.2. If it rains, the probability that Mia is late is 0.3; if it is dry, it is 0.1. Work out the probability that Mia is late. [2]



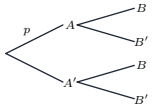
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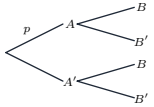
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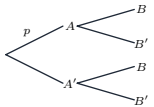
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3.2

- $P(A) \times P(B) = 0.4 \times 0.5 = 0.2$
- this equals $P(A \cap B)$, so **yes**, they are independent

Full solution on next slide

Full solution 3.2

3.2

- $P(A) \times P(B) = 0.4 \times 0.5 = 0.2$
- this equals $P(A \cap B)$, so **yes**, they are independent

3.3

- $P(\text{both black}) = \frac{6}{10} \times \frac{5}{9} = \frac{30}{90}$ and $P(\text{both white}) = \frac{4}{10} \times \frac{3}{9} = \frac{12}{90}$
- $P(\text{same}) = \frac{30}{90} + \frac{12}{90} = \frac{42}{90}$
- $P(\text{both black} \mid \text{same}) = \dfrac{30}{42} = \mathbf{\dfrac{5}{7}}$

3.4

- rain and late $= 0.2 \times 0.3 = 0.06$; dry and late $= 0.8 \times 0.1 = 0.08$
- $P(\text{late}) = 0.06 + 0.08 = \mathbf{0.14}$

3.5

- given late, only the two "late" routes count, total 0.14
- $P(\text{rain} \mid \text{late}) = \dfrac{P(\text{rain and late})}{P(\text{late})} = \dfrac{0.06}{0.14} = \mathbf{\dfrac{3}{7}}$