Exercise walk-through

5.2.1
Event Sets

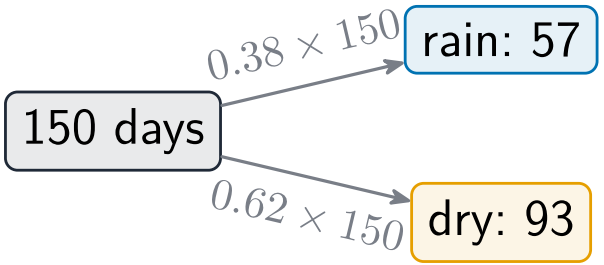
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Experiments and expected frequency

- When outcomes are not **equally likely**, such as a drawing pin landing point up, we estimate the probability by experiment.
- The **relative frequency** is
- A larger **sample size** gives a better estimate.
- Six heads in 10 tosses gives 0.6, but 502 heads in 1000 tosses gives 0.502, much closer to the true 0.5 of a fair coin.

$$\text{relative frequency} = \frac{\text{number of times the event happened}}{\text{number of trials}}$$

Figure: Experiments and expected frequency (2)



expected counts: a long-run average, not a guarantee

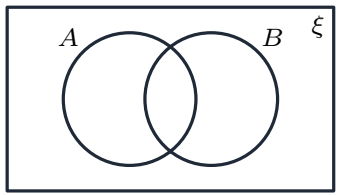
- A **frequency tree** with expected counts: 57 rainy and 93 dry days out of 150
- Match each part of the diagram to the text.

Sources for this lesson

- AQA, *GCSE Mathematics (8300) specification*, subject content section 3.5 (Probability).
- M. vos Savant, "Ask Marilyn", *Parade* (9 September 1990), and her follow-up columns of December 1990 and February 1991.
- AQA, *GCSE Mathematics* Higher Papers 2 and 3 (June 2024), the expected-frequency, dice and machine questions, adapted.

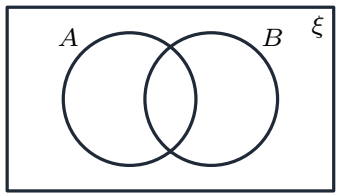
Exercise 2.1

2.1 Use the worked example. Work out the probability that a student chosen at random swims only. [1]



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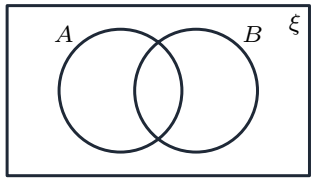


2.1

- $\frac{5}{30} = \mathbf{\frac{1}{6}}$

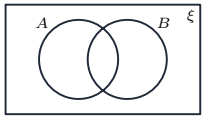
Setup for exercise 2.2

2.2 Of 40 students, 22 are boys. 9 boys and 6 girls wear glasses. How many girls do not wear glasses? [1]

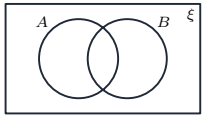


Exercise 2.2

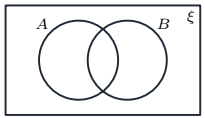
2.3 $\xi = \{1, 2, 3, \ldots, 10\}$, A = even numbers and B = multiples of 3. List $A \cap B$ and $A \cup B$. [2]



2.4 Of 50 people, 30 like tea, 25 like coffee and 12 like both. How many like neither? [2]

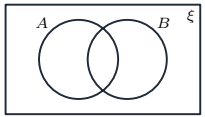


2.5 For the sets in **2.3**, work out $P(A')$ for a number chosen at random from ξ. [1]

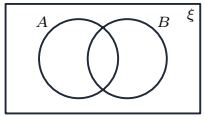


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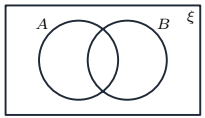
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2.2

- girls $= 40 - 22 = 18$, so $18 - 6 =$ **12**

Full solution on next slide

Full solution 2.2

2.2

- girls $= 40 - 22 = 18$, so $18 - 6 = \mathbf{12}$

2.3

- $A \cap B = \{\mathbf{6}\}$
- $A \cup B = \{\mathbf{2}, \mathbf{3}, \mathbf{4}, \mathbf{6}, \mathbf{8}, \mathbf{9}, \mathbf{10}\}$

2.4

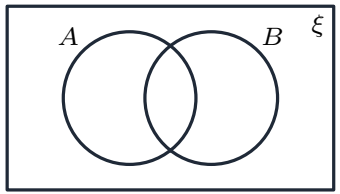
- at least one $= 30 + 25 - 12 = 43$
- neither $= 50 - 43 = \mathbf{7}$

2.5

- $A' = \{1, 3, 5, 7, 9\}$: $\dfrac{5}{10} = \mathbf{\dfrac{1}{2}}$

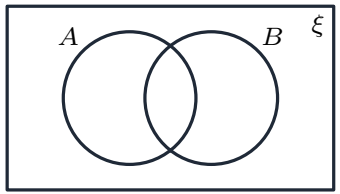
Exercise 3.1

3.1 Ravi says that $30 + 25 = 55$ people in **2.4** like tea or coffee. Explain his mistake. [1]



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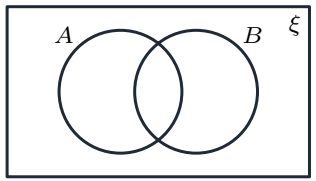


3.1

- the 12 who like both are counted twice; it should be $30 + 25 - 12 = 43$

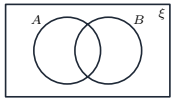
Setup for exercise 3.2

3.2 80 students; $\frac{3}{5}$ of them walk to school. $\frac{1}{4}$ of the walkers are late, and 6 of the others are late. Use a frequency tree to work out the probability that a student chosen at random is late. [3]

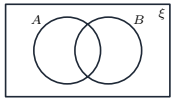


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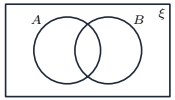
3.3 $\xi = \{1, 2, \ldots, 12\}$, $P =$ prime numbers and $Q =$ odd numbers. List $P \cap Q'$, and work out the probability that a number chosen at random is in $Q \cap P'$. [3]



3.4 Of 60 people, 35 have a dog, 28 have a cat and 10 have neither. Work out how many have both, and the probability that a person chosen at random has a dog but not a cat. [3]

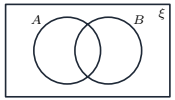


3.5 120 students each choose French or Spanish. The ratio of boys to girls is 5 : 7. 30 boys choose French, and $\frac{3}{7}$ of the girls choose Spanish. Work out the probability that a student chosen at random chooses Spanish. [4]

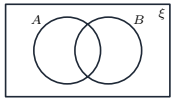


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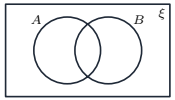
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3.2

- walkers $= \frac{3}{5} \times 80 = 48$; late walkers $= \frac{1}{4} \times 48 = 12$
- late $= 12 + 6 = 18$

Full solution on next slide

Full solution 3.2 (1/2)

3.2

- walkers $= \frac{3}{5} \times 80 = 48$; late walkers $= \frac{1}{4} \times 48 = 12$
- late $= 12 + 6 = 18$
- $P(\text{late}) = \dfrac{18}{80} = \dfrac{\mathbf{9}}{\mathbf{40}}$

3.3

- $P = \{2, 3, 5, 7, 11\}$ and $Q = \{1, 3, 5, 7, 9, 11\}$; $P \cap Q' = \{\mathbf{2}\}$
- $Q \cap P' = \{1, 9\}$
- probability $= \dfrac{2}{12} = \dfrac{\mathbf{1}}{\mathbf{6}}$

Full solution 3.2 (2/2)

3.4

- at least one $= 60 - 10 = 50$, so both $= 35 + 28 - 50 = \mathbf{13}$
- dog only $= 35 - 13 = 22$: $\dfrac{22}{60} = \dfrac{\mathbf{11}}{\mathbf{30}}$

3.5

- boys $= \frac{5}{12} \times 120 = 50$ and girls $= 70$
- boys choosing Spanish $= 50 - 30 = 20$; girls choosing Spanish $= \frac{3}{7} \times 70 = 30$
- Spanish $= 20 + 30 = 50$
- $P(\text{Spanish}) = \dfrac{50}{120} = \dfrac{\mathbf{5}}{\mathbf{12}}$